## Feature Tracking and Video Textures

## Feature Tracking

Why is motion of features useful?


## Feature Tracking

## Why is motion of features useful?



Slide credit: Niebles and Krishna

## Estimating Optical Flow




Given two subsequent frames, estimate the apparent motion field $u(x, y), v(x, y)$ between them

## Key assumptions

- Brightness constancy: projection of the same point looks the same in every frame
- Small motion: points do not move very far
- Spatial coherence: points move like their neighbors


## The brightness constancy constraint



Brightness Constancy Equation:

$$
I(x, y, t-1)=I(x+u(x, y), y+v(x, y), t)
$$

Linearizing the right side using Taylor expansion:

$$
\begin{aligned}
& I(x+u, y+v, t) \approx I(x, y, t-1)+I_{x} u(x, y)+I_{y} \cdot v(x, y)+I_{t} \\
& I(x+u, y+v, t)-I(x, y, t-1)=I_{x} \cdot u(x, y)+I_{y} \cdot v(x, y)+I_{t}
\end{aligned}
$$

Hence, $I_{x} \cdot u+I_{y} \cdot v+I_{t} \approx 0 \rightarrow \nabla I \cdot\left[\begin{array}{ll}u & v\end{array}\right]^{T}+I_{t}=0$

## Computing Derivatives

$$
\begin{aligned}
& {\left[\begin{array}{ll}
-1 & 1 \\
-1 & 1
\end{array}\right] \text { first image } \quad\left[\begin{array}{cc}
-1 & -1 \\
1 & 1
\end{array}\right] \text { first image } \quad\left[\begin{array}{ll}
-1 & -1 \\
-1 & -1
\end{array}\right] \text { first image }} \\
& {\left[\begin{array}{ll}
-1 & 1 \\
-1 & 1
\end{array}\right] \text { second image }\left[\begin{array}{cc}
-1 & -1 \\
1 & 1
\end{array}\right] \text { second image }\left[\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right] \text { second image }} \\
& I_{x} \\
& I_{y} \\
& I_{t}
\end{aligned}
$$

## The brightness constancy constraint

Can we use this equation to recover image motion ( $u, v$ ) at each pixel?

$$
\nabla I \cdot\left[\begin{array}{ll}
u & v
\end{array}\right]^{T}+I_{t}=0
$$

How many equations and unknowns per pixel?
One equation (this is a scalar equation!), two unknowns ( $u, v$ )

The component of the flow perpendicular to the gradient (i.e., parallel to an edge) cannot be measured


## The aperture problem



## The aperture problem



Slide credit: Savarese

## Solving the Ambiguity

How to get more equations for a pixel?

## Spatial coherence constraint:

Assume the pixel's neighbors have the same (u,v)
If we use a $5 \times 5$ window, that gives us 25 equations per pixel

$$
\begin{gathered}
0=I_{t}\left(\mathbf{p}_{\mathbf{i}}\right)+\nabla I\left(\mathbf{p}_{\mathbf{i}}\right) \cdot\left[\begin{array}{ll}
u & v
\end{array}\right] \\
{\left[\begin{array}{cc}
I_{x}\left(\mathbf{p}_{1}\right) & I_{y}\left(\mathbf{p}_{1}\right) \\
I_{x}\left(\mathbf{p}_{2}\right) & I_{y}\left(\mathbf{p}_{2}\right) \\
\vdots & \vdots \\
I_{x}\left(\mathbf{p}_{25}\right) & I_{y}\left(\mathbf{p}_{25}\right)
\end{array}\right]\left[\begin{array}{c}
u \\
v
\end{array}\right]=-\left[\begin{array}{c}
I_{t}\left(\mathbf{p}_{1}\right) \\
I_{t}\left(\mathbf{p}_{2}\right) \\
\vdots \\
I_{t}\left(\mathbf{p}_{25}\right)
\end{array}\right]}
\end{gathered}
$$

## Lucas-Kanade Flow

Overconstrained linear system:
\(\left[$$
\begin{array}{cc}I_{x}\left(\mathbf{p}_{1}\right) & I_{y}\left(\mathbf{p}_{1}\right) \\
I_{x}\left(\mathbf{p}_{2}\right) & I_{y}\left(\mathbf{p}_{2}\right) \\
\vdots & \vdots \\
I_{x}\left(\mathbf{p}_{\mathbf{2 5}}\right) & I_{y}\left(\mathbf{p}_{\mathbf{2 5}}\right)\end{array}
$$\right]\left[$$
\begin{array}{l}u \\
v\end{array}
$$\right]=-\left[\begin{array}{c}I_{t}\left(\mathbf{p}_{1}\right) <br>
I_{t}\left(\mathbf{p}_{2}\right) <br>
\vdots <br>

I_{t}\left(\mathbf{p}_{\mathbf{2 5}}\right)\end{array}\right]\)| $A$ | $d=b$ |
| :---: | :---: |
| $25 \times 2$ | $2 \times 1$ |
| $25 \times 1$ |  |

## Lucas-Kanade Flow

Overconstrained linear system:

$$
\left[\begin{array}{cc}
I_{x}\left(\mathbf{p}_{1}\right) & I_{y}\left(\mathrm{p}_{1}\right) \\
I_{x}\left(\mathrm{p}_{2}\right) & I_{y}\left(\mathrm{p}_{2}\right) \\
\vdots & \vdots \\
I_{x}\left(\mathrm{p}_{25}\right) & I_{y}\left(\mathrm{p}_{25}\right)
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right]=-\left[\begin{array}{c}
I_{t}\left(\mathrm{p}_{1}\right) \\
I_{t}\left(\mathrm{p}_{2}\right) \\
\vdots \\
I_{t}\left(\mathbf{p}_{25}\right)
\end{array}\right] \quad \begin{gathered}
A \\
\hline 2 \times 2
\end{gathered} \quad d=b
$$

Least squares solution for $\boldsymbol{d}$ given by $\left(A^{T} A\right) d=A^{T} b$

$$
\begin{gathered}
{\left[\begin{array}{cc}
\sum I_{x} I_{x} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y} I_{y}
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right]=-\left[\begin{array}{c}
\sum I_{x} I_{t} \\
\sum I_{y} I_{t}
\end{array}\right]} \\
A^{T} A
\end{gathered}
$$

The summations are over all pixels in the $5 \times 5$ window

## Conditions for Solvability

Optimal ( $\mathbf{u}, \mathrm{v}$ ) satisfies Lucas-Kanade equation

$$
\begin{array}{cc}
{\left[\begin{array}{cc}
\sum I_{x} I_{x} & \sum I_{x} I_{y} \\
\sum I_{x} I_{y} & \sum I_{y} I_{y}
\end{array}\right]\left[\begin{array}{l}
u \\
v
\end{array}\right]=-\left[\begin{array}{c}
\sum I_{x} I_{t} \\
\sum I_{y} I_{t}
\end{array}\right]} \\
A^{T} A & A^{T} b
\end{array}
$$

When is this Solvable?

- $A^{\top} A$ should be invertible
- $A^{\top} A$ should not be too small due to noise
- eigenvalues $\lambda_{1}$ and $\lambda_{2}$ of $\mathbf{A}^{\top} \mathbf{A}$ should not be too small
- $\mathbf{A}^{\mathrm{T}} \mathrm{A}$ should be well-conditioned
- $\lambda_{1} / \lambda_{2}$ should not be too large ( $\lambda_{1}=$ larger eigenvalue)


## $M=\mathrm{A}^{\mathrm{T}} \mathrm{A}$ is the second moment matrix ! (Harris corner detector...)

- Eigenvectors and eigenvalues of $A^{\top} A$ relate to edge direction and magnitude
- The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change
- The other eigenvector is orthogonal to it


## Edge


$\sum \nabla I(\nabla I)^{T}$

- gradients very large or very small
- large $\lambda_{1}$, small $\lambda_{2}$
$\qquad$


## Low Texture Region


$\sum \nabla I(\nabla I)^{T}$

- gradients have small magnitude
- small $\lambda_{1}$, small $\lambda_{2}$


## High Texture Region


$\sum \nabla I(\nabla I)^{T}$

- gradients are different, large magnitudes - large $\lambda_{1}$, large $\lambda_{2}$


## Revisiting Small Motion Assumption



Is this motion small enough?
Probably not-it's much larger than one pixel (2 $2^{\text {nd }}$ order terms dominate) How might we solve this problem?

## Reduce Resolution



## Course to Fine Estimation



Gaussian pyramid of image 1

# Course to Fine Estimation 



## Optical Flow Results



## Optical Flow Results



Video Puppetry: A Performative Interface for Cutout Animation. Connelly Barnes, David E. Jacobs, Jason Sanders, Dan B Goldman, Szymon Rusinkiewicz, Adam Finkelstein and Maneesh Agrawala, SIGGRAPH ASIA 2008.


## Identification and Tracking: SIFT



+ Identifies and locates puppets
- Not real time


## Identification and Tracking: KLT



+ Real time
- No identification


## Identification and Tracking: SIFT + KLT



Group KLT points by puppet
Update transform from KLT motion
Use SIFT to correct KLT drift


## Video Textures

Video Textures. Arno Schoedl, Richard Szeliski, David Salesin and Irfan Essa, SIGGRAPH 2000.

## Weather Forecasting for Dummies ${ }^{\text {TM }}$

## Let's predict weather:

- Given today's weather only, we want to know tomorrow's
- Suppose weather can only be \{Sunny, Cloudy, Raining\}


## The "Weather Channel" algorithm:

- Over a long period of time, record:
- How often S followed by R
- How often S followed by S
- Etc.
- Compute percentages for each state:
- $P(R \mid S), P(S \mid S)$, etc.
- Predict the state with highest probability!
- It's a Markov Chain


## Markov Chain



## Text Synthesis

[Shannon,'48] proposed a way to generate English-looking text using N-grams:

- Assume a generalized Markov model
- Use a large text to compute prob. distributions of each letter given N-1 previous letters
- Starting from a seed repeatedly sample this Markov chain to generate new letters
- Also works for whole words


## Mark V. Shaney (Bell Labs)

Results (using alt. singles corpus):

- "As I've commented before, really relating to someone involves standing next to impossible."
- "One morning I shot an elephant in my arms and kissed him."
- "I spent an interesting evening recently with a grain of salt"


## Still Photos



## Video Clips



## Video Textures



## Problem Statement


video clip

video texture

## Our Approach



How do we find good transitions?

## Finding Good Transitions

Compute $L_{2}$ distance $D_{i, j}$ between all frames


Similar frames make good transitions

## Markov Chain Representation



Similar frames make good transitions

## Transition Costs

Transition from $i$ to $j$ if successor of $i$ is similar to $j$
Cost function: $C_{i \rightarrow j}=D_{i+1, j}$


## Transition Probabilities

Probability for transition $\mathrm{P}_{\mathrm{i} \rightarrow \mathrm{j}}$ inversely related to cost:

$$
P_{i \rightarrow j} \sim \exp \left(-C_{i \rightarrow j} / \sigma^{2}\right)
$$


high $\sigma$

low $\sigma$

## Preserving Dynamics



## Preserving Dynamics



## Preserving Dynamics

Cost for transition $i \rightarrow j$

$$
C_{i \rightarrow j}=\sum_{k=-\mathrm{N}}^{\mathrm{N}-1} w_{k} D_{i+k+1, j+k}
$$



## Preserving Dynamics - Effect

Cost for transition $i \rightarrow j$

$$
C_{i \rightarrow j}=\sum_{k=-\mathrm{N}}^{\mathrm{N}-1} w_{k} D_{i+k+1, j+k}
$$



## Dead Ends

No good transition at the end of sequence


## Future Cost

- Propagate future transition costs backward
- Iteratively compute new cost



## Future Cost

- Propagate future transition costs backward
- Iteratively compute new cost



## Future Cost

- Propagate future transition costs backward
- Iteratively compute new cost



## Future Cost

- Propagate future transition costs backward
- Iteratively compute new cost



## Future Cost

- Propagate future transition costs backward
- Iteratively compute new cost

- Q-learning


## Future Cost - Effect



## Visual Discontinuities

Problem: Visible "Jumps"


## Crossfading

Solution: Crossfade from one sequence to the other

$$
\begin{aligned}
& \ldots \begin{array}{l:l:l:l}
\hline A_{i-2} & \frac{3}{4} & A_{i-1} & \frac{2}{4} \\
\hline A_{i} & \frac{1}{4} & A_{i+1} \\
\hline
\end{array} \\
& +\frac{1}{4} \begin{array}{|c:c:c}
\mathrm{B}_{\mathrm{j}-2} & +\frac{2}{4} \mathrm{~B}_{\mathrm{j}-1} & +\frac{3}{4} \mathrm{~B}_{\mathrm{j}} \\
\hline
\end{array} \\
& B_{i+1} \ldots \\
& \begin{array}{|l|l:l:l|l|l|}
\hline \mathrm{A}_{\mathrm{i}-2} & \mathrm{~A}_{\mathrm{i}-1} / \mathrm{B}_{\mathrm{i}-2} & \mathrm{~A}_{\mathrm{i}-1} / \mathrm{B}_{\mathrm{j}-2} & \mathrm{~A}_{\mathrm{i}-1} / \mathrm{B}_{\mathrm{j}-2} & \mathrm{~B}_{\mathrm{j}+1} \\
\hline
\end{array}
\end{aligned}
$$

## Crossfading



## Frequent Jump \& Crossfading



## Video Portrait



Useful for web pages

## Video Portrait - 3D



Combine with IBR techniques

## Region-Based Analysis

Divide video up into regions


Generate a video texture for each region

## Automatic Region Analysis



What if motion regions overlap in space?

## User-Controlled Video Textures


slow

variable

fast

User selects target frame range

## Time Warping



Lengthen / shorten video without affecting speed

## Video-Based Animation

Like sprites computer games
Extract sprites from real video
Interactively control desired motion


## Video Sprite Extraction


blue screen matting
and velocity estimation


## Video Sprite Control

Augmented transition cost:


## Video Sprite Control

Need future cost computation
Precompute future costs for a few angles.
Switch between precomputed angles according to user input
[GIT-GVU-00-11]


## Interactive Fish



What would be required to create video sprite of a human?

## Panoramic Video Textures



Panoramic Video Textures. Aseem Agarwala, Ke Colin Zheng, Chris Pal, Maneesh Agrawala, Michael F. Cohen, Brian Curless, David Salesin, Richard Szeliski. SIGGRAPH 2005.

## "Amateur" by Lasse Gjertsen

http://www.youtube.com/watch?v=JzqumbhfxRo

## Michel Gondry Train Video

https://www.youtube.com/watch?v=0S43IwBF0uM

