

# Network Layout

*Maneesh Agrawala*

**CS 448B: Visualization**  
**Winter 2020**

1

# Announcements

3

# Final project

---

## New visualization research or data analysis project

- **Research:** Pose problem, Implement creative solution
- **Data analysis:** Analyze dataset in depth & make a visual explainer

## Deliverables

- **Research:** Implementation of solution
- **Data analysis/explainer:** Article with multiple interactive visualizations
- 6-8 page paper

## Schedule

- Project proposal: **Wed 2/19**
- Design review and feedback: **3/9 and 3/11**
- Final presentation: **3/16 (7-9pm) Location: TBD**
- Final code and writeup: **3/18 11:59pm**

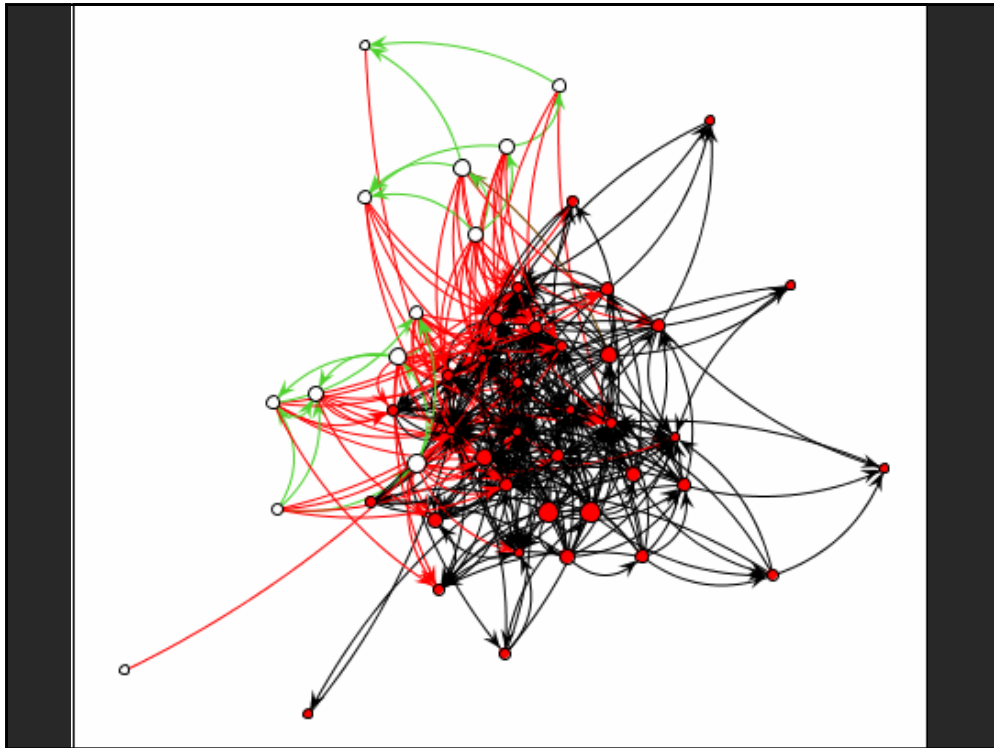
## Grading

- Groups of **up to 3 people**, graded individually
- Clearly report responsibilities of each member

4

# Network Layout

5



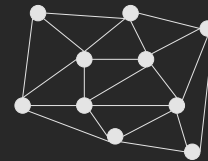
6

## Graphs and Trees

### Graphs

Model relations among data

Nodes and edges

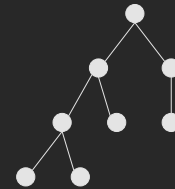


### Trees

Graphs with hierarchical structure

Connected graph with  $N-1$  edges

Nodes as *parents* and *children*



7

# Spatial Layout

---

**Primary concern – layout of nodes and edges**

**Often (but not always) goal is to depict structure**

- Connectivity, path-following
- Network distance
- Clustering
- Ordering (e.g., hierarchy level)

8

# Applications

---

Tournaments

Organization Charts

Genealogy

Diagramming (e.g., Visio)

Biological Interactions (Genes, Proteins)

Computer Networks

Social Networks

Simulation and Modeling

Integrated Circuit Design

9



# Topics

---

Tree Layout

Network Layout

    Sugiyama-Style Layout

    Force-Directed Layout

Alternatives to Network Layout

    Matrix Diagrams

    Attribute-Drive Layout

10

# Tree Layout

11

# Tree Visualization

## Indentation

- Linear list, indentation encodes depth



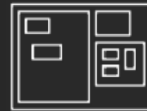
## Node-Link diagrams

- Nodes connected by lines/curves



## Enclosure diagrams

- Represent hierarchy by enclosure



## Layering

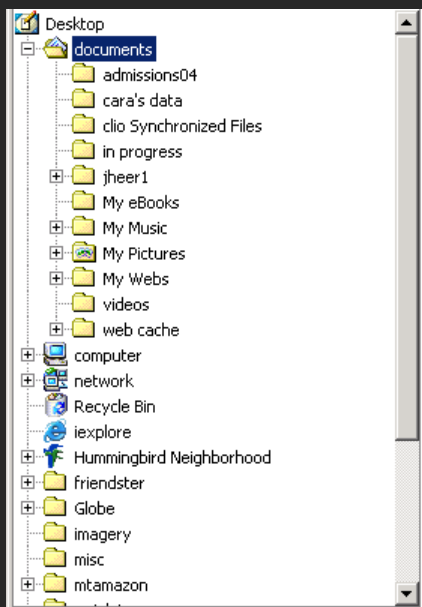
- Layering and alignment



**Tree layout is fast:  $O(n)$  or  $O(n \log n)$ , enabling real-time layout for interaction**

12

# Indentation



Items along vertically spaced rows

Indentation shows parent/child relationships

Often used in interfaces

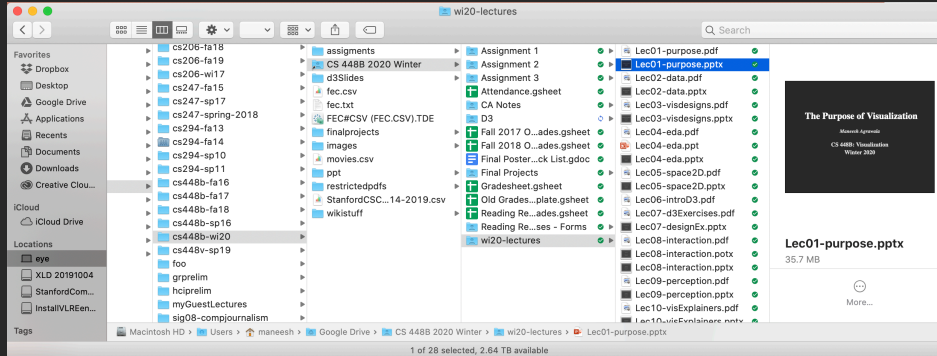
Breadth/depth contend for space

Often requires scrolling



13

# Single-Focus (Accordion) List



Separate breadth & depth in 2D  
Focus on single path at a time

14

# Node-Link Diagrams

Nodes distributed in space, connected by lines  
Use 2D space to break apart breadth and depth  
Space used to communicate hierarchical orientation  
Typically towards authority or generality

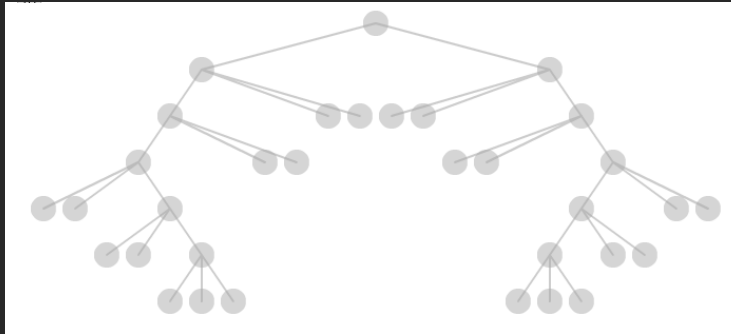


15

# Basic Recursive Approach

Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension



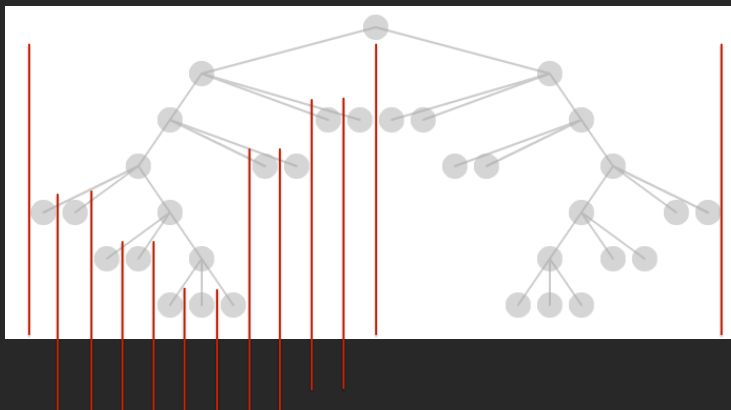
17

# Basic Recursive Approach

Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension

Problem:



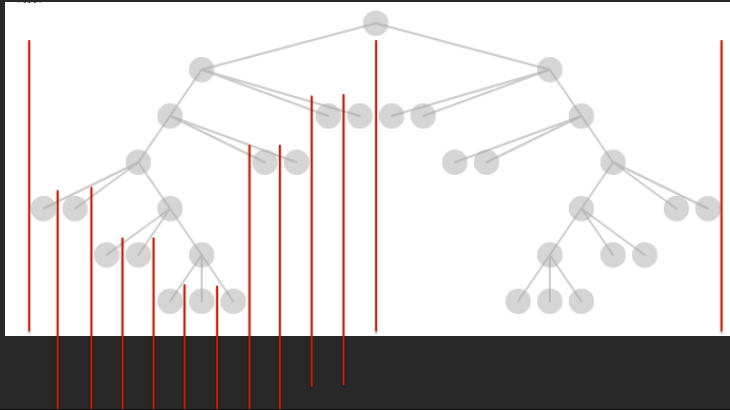
18

# Basic Recursive Approach

Repeatedly divide space for subtrees by leaf count

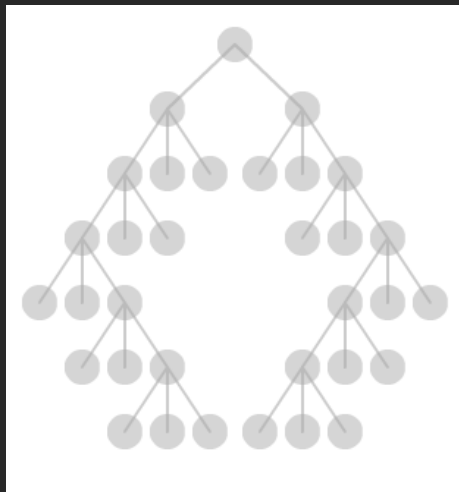
- Breadth of tree along one dimension
- Depth along the other dimension

Problem: Exponential growth of breadth



19

# Reingold & Tilford's Tidier Layout



**Goal:** maximize density and symmetry.

Originally for binary trees, extended by Walker to cover general case.

This extension was corrected by Buchheim et al. to achieve a linear time algorithm

20

# Reingold-Tilford Layout

---

## Design concerns

- Clearly encode depth level
- No edge crossings
- Isomorphic subtrees drawn identically
- Ordering and symmetry preserved
- Compact layout (don't waste space)*

21

# Reingold-Tilford Algorithm

---

**Linear algorithm – starts with bottom-up (postorder) pass**

**Set Y-coord by depth, arbitrary starting X-coord**

**Merge left and right subtrees**

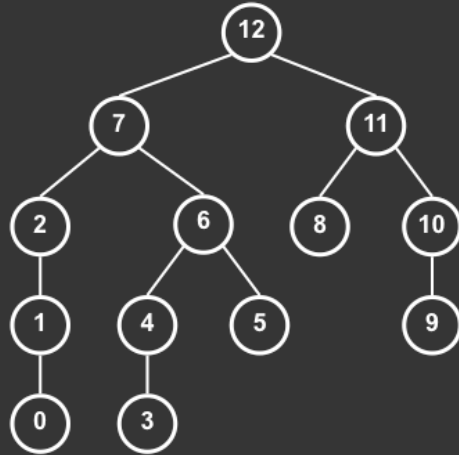
- Shift right as close as possible to left
  - Computed efficiently by maintaining subtree contours
- “Shifts” in position saved for each node as visited
- Parent nodes are centered above their children

**Top-down (preorder) pass for assignment of final positions**

- Sum of initial layout and aggregated shifts

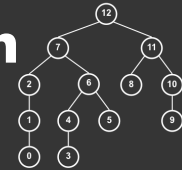
22

# Reingold-Tilford Algorithm



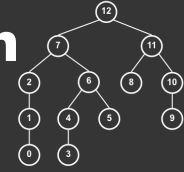
23

# Reingold-Tilford Algorithm



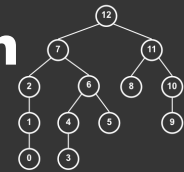
24

# Reingold-Tilford Algorithm



25

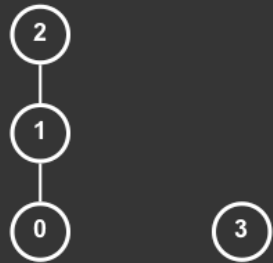
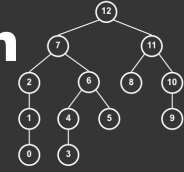
# Reingold-Tilford Algorithm



26

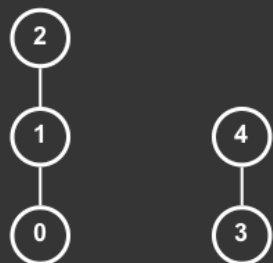
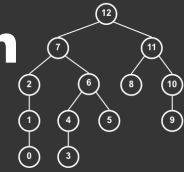


# Reingold-Tilford Algorithm



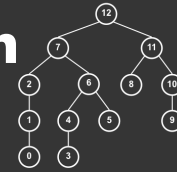
27

# Reingold-Tilford Algorithm



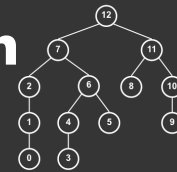
28

# Reingold-Tilford Algorithm



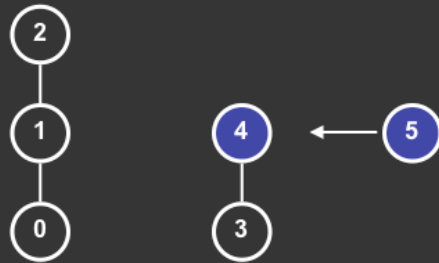
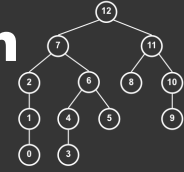
29

# Reingold-Tilford Algorithm



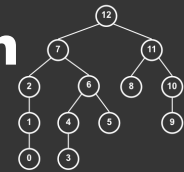
30

# Reingold-Tilford Algorithm



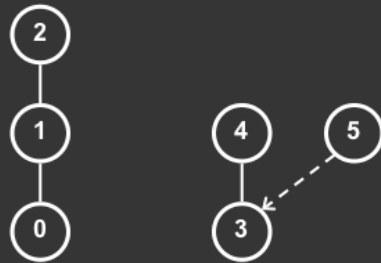
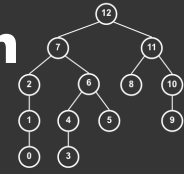
31

# Reingold-Tilford Algorithm



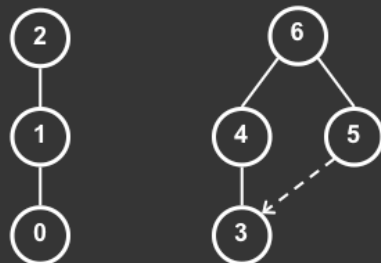
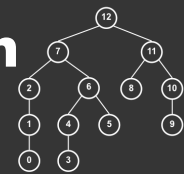
32

# Reingold-Tilford Algorithm



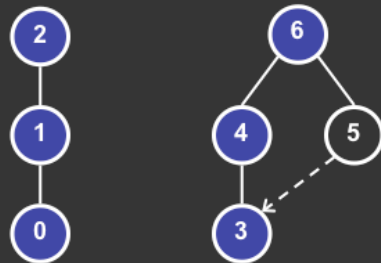
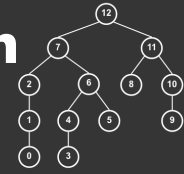
33

# Reingold-Tilford Algorithm



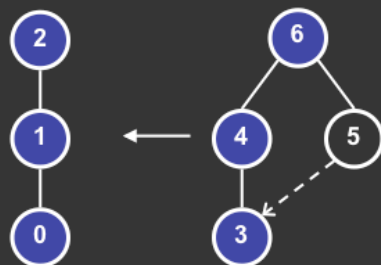
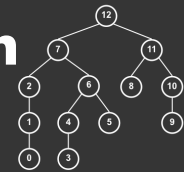
34

# Reingold-Tilford Algorithm



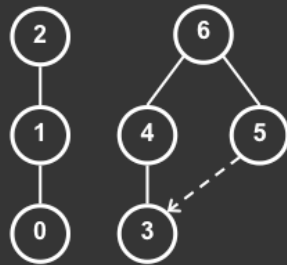
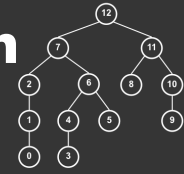
35

# Reingold-Tilford Algorithm



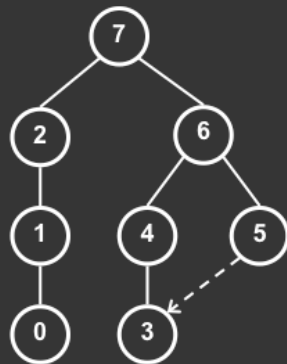
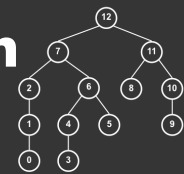
36

# Reingold-Tilford Algorithm



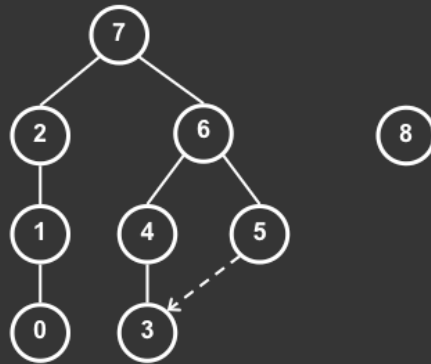
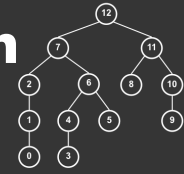
37

# Reingold-Tilford Algorithm



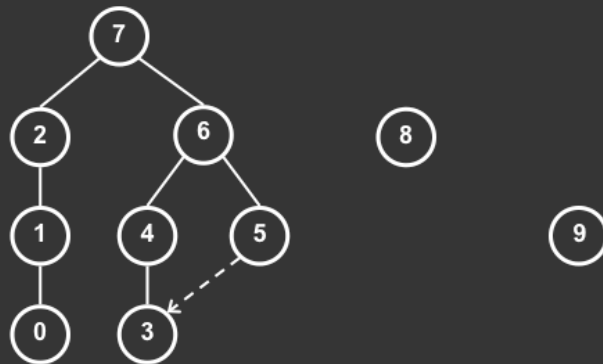
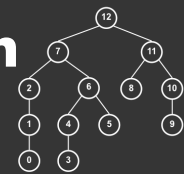
38

# Reingold-Tilford Algorithm



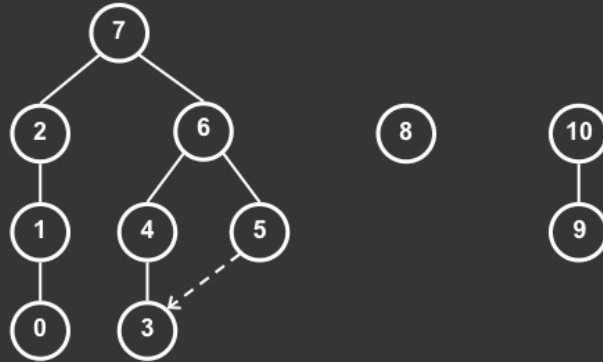
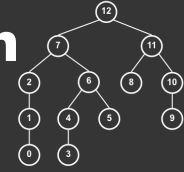
39

# Reingold-Tilford Algorithm



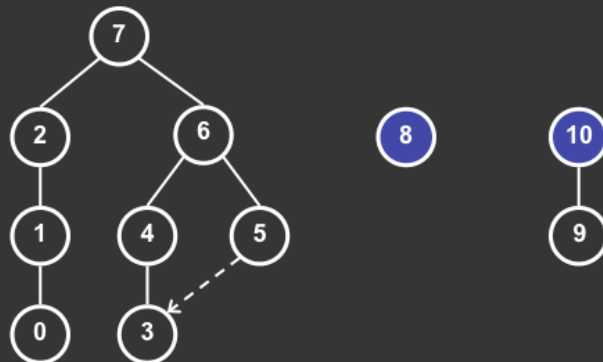
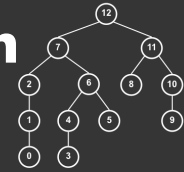
40

# Reingold-Tilford Algorithm



41

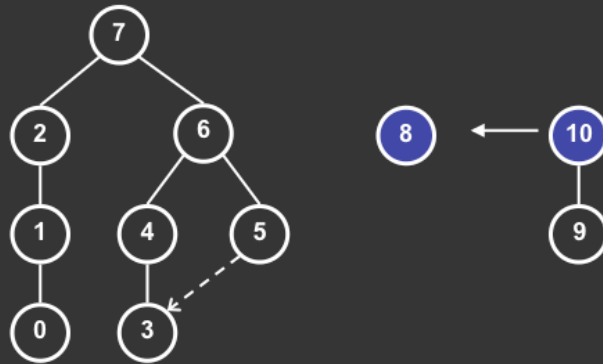
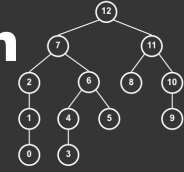
# Reingold-Tilford Algorithm



42

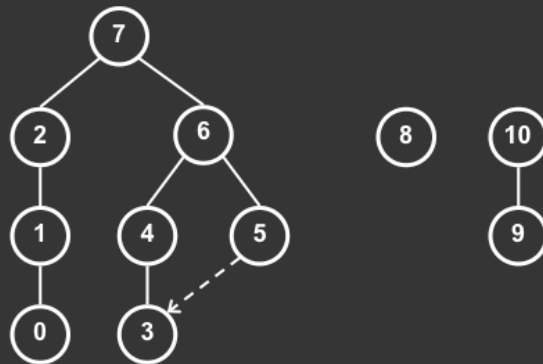
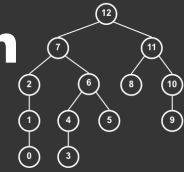


# Reingold-Tilford Algorithm



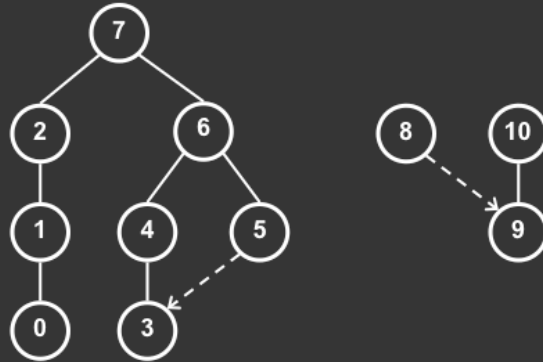
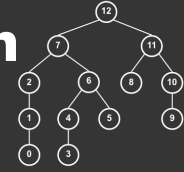
43

# Reingold-Tilford Algorithm



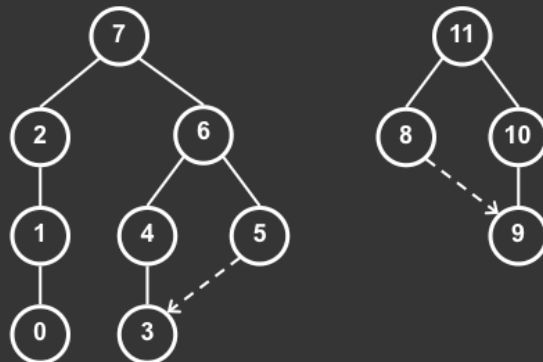
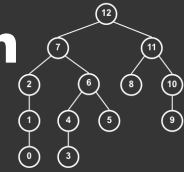
44

# Reingold-Tilford Algorithm



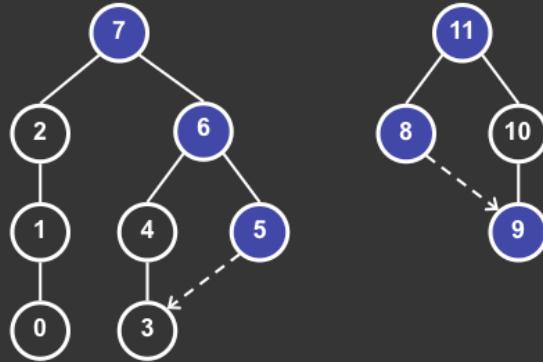
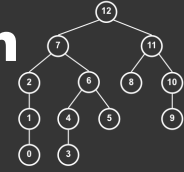
45

# Reingold-Tilford Algorithm



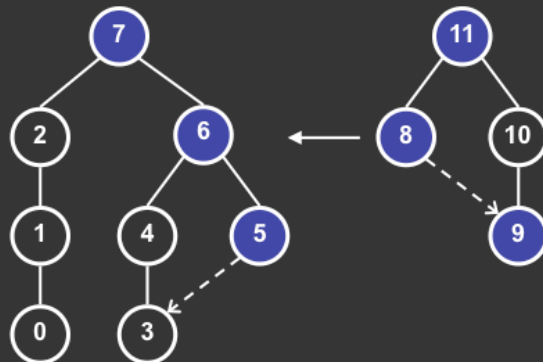
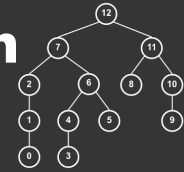
46

# Reingold-Tilford Algorithm



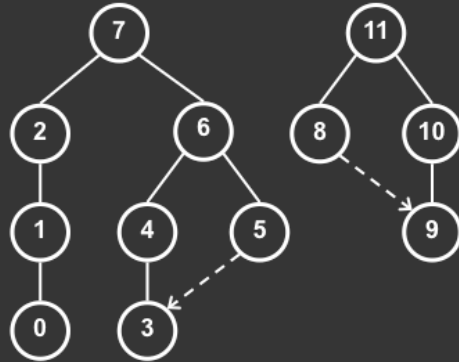
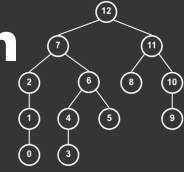
47

# Reingold-Tilford Algorithm



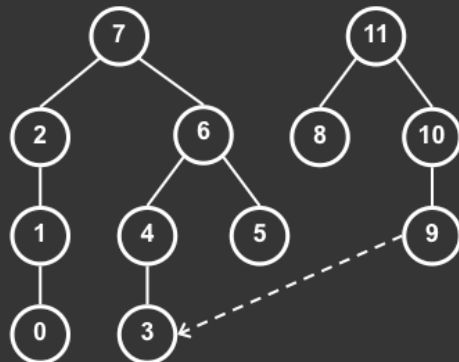
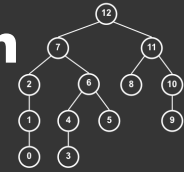
48

# Reingold-Tilford Algorithm



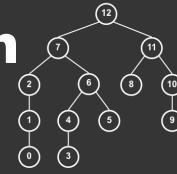
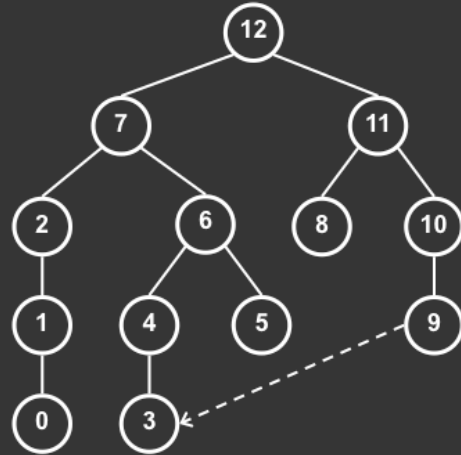
49

# Reingold-Tilford Algorithm



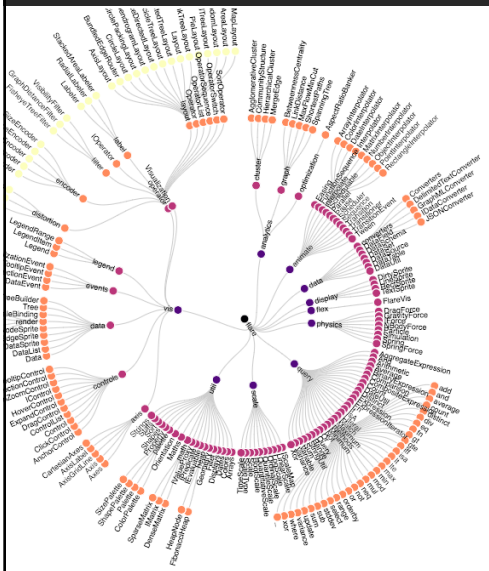
50

# Reingold-Tilford Algorithm



51

# Radial Layout



Node-link diagram in polar coords

Radius encodes depth root at center

Angular sectors assigned to subtrees (recursive approach)

Reingold-Tilford approach can also be applied here

54

# Problems with Node-Link Diagrams

## Scale

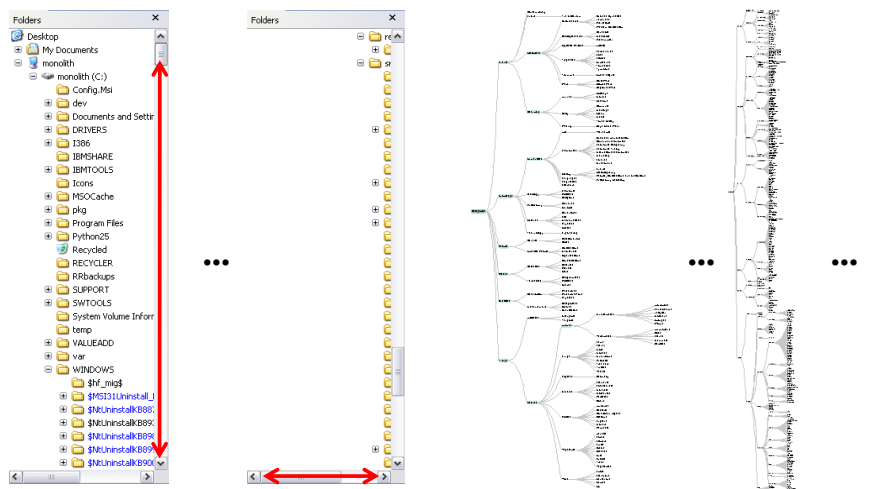
Tree breadth often grows exponentially  
Even with tidier layout, quickly run out of space

## Possible solutions

- Filtering
- Focus+Context
- Scrolling or Panning
- Zooming
- Aggregation

57

# Visualizing Large Hierarchies



Indented Layout

Reingold-Tilford Layout

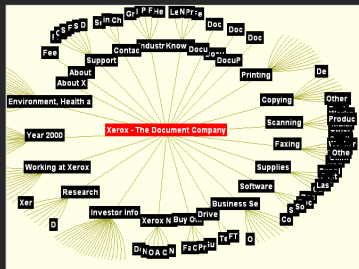
58



MC Escher, Circle Limit IV

59

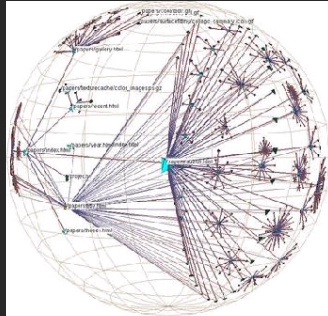
# Hyperbolic Layout



Layout in hyperbolic space, then project on to Euclidean plane

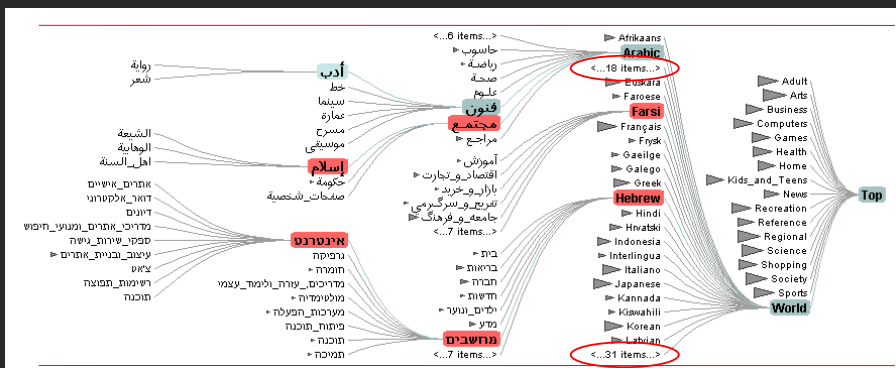
Why? Like tree breadth, the hyperbolic plane expands exponentially

Also computable in 3D, projected into a sphere



60

# Degree-of-Interest Trees [AVI 04]



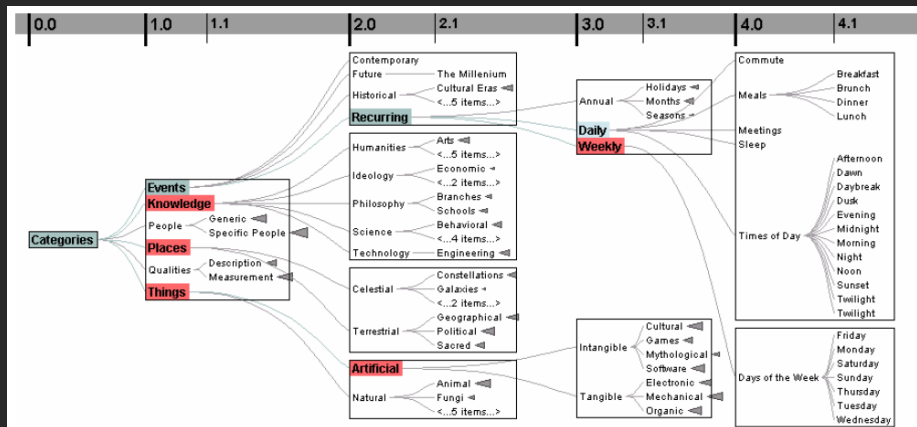
Space-constrained, multi-focal tree layout

<https://www.youtube.com/watch?v=RTQ0N4QY0yc>

<https://observablehq.com/@d3/collapsible-tree>

61

# Degree-of-Interest Trees



Cull “un-interesting” nodes on a per block basis until all blocks on a level fit within bounds

Center child blocks under parents

<https://www.youtube.com/watch?v=RTQ0N4QY0yc>

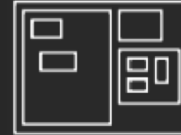
<https://observablehq.com/@d3/collapsible-tree>

62



# Enclosure Diagrams

Encode structure using spatial enclosure  
Popularly known as TreeMaps



## Benefits

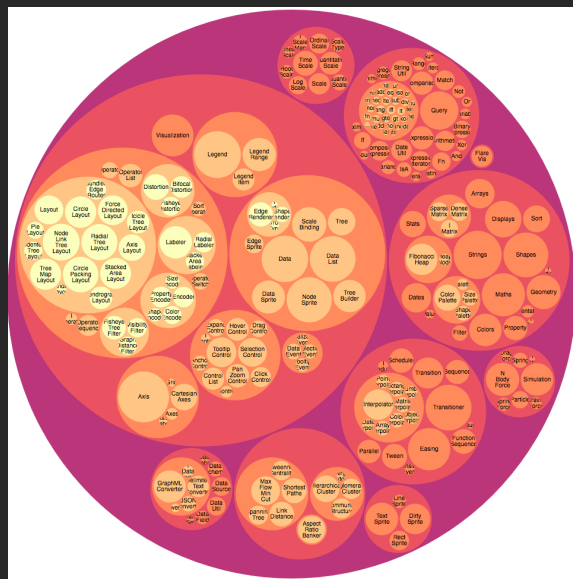
- Provides a single view of an entire tree
- Easier to spot large/small nodes

## Problems

- Difficult to accurately read depth

63

# Circle Packing Layout



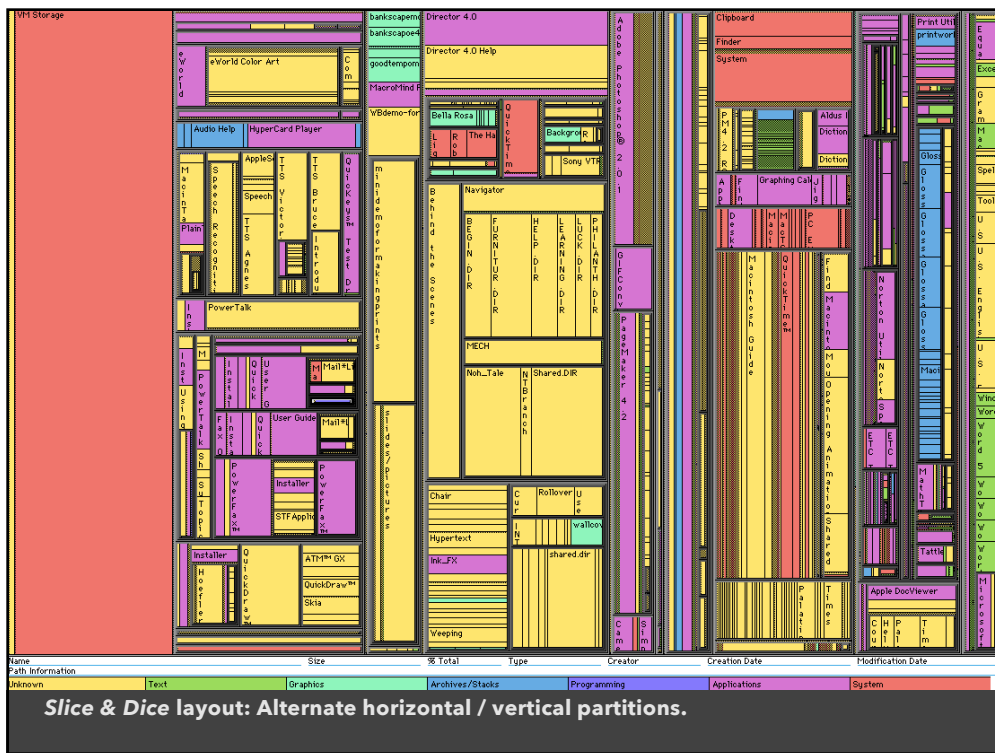
Nodes represented as sized circles

Nesting to show parent-child relationships

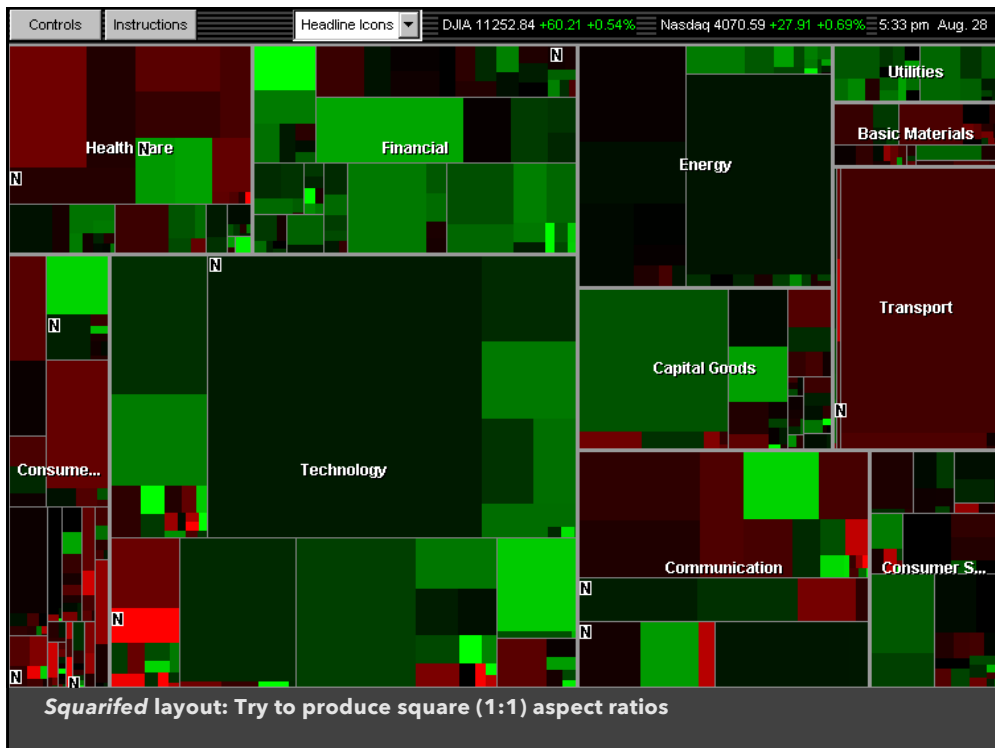
**Problems:**

64





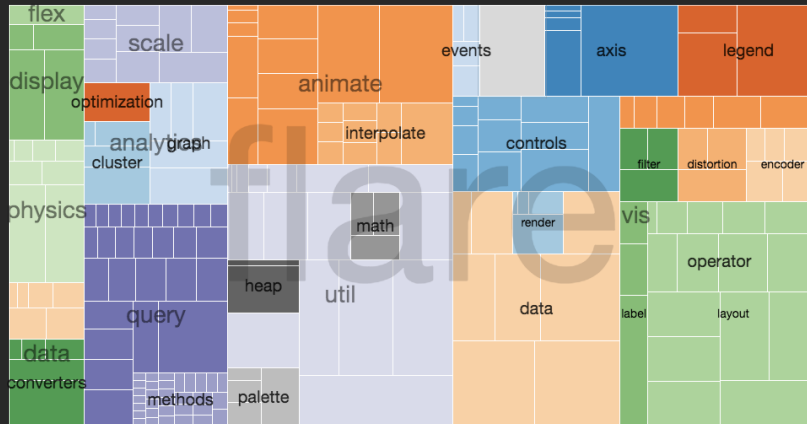
68



69

# Squarified Treemaps [Bruls 00]

Greedy optimization for objective of square rectangles  
Slice/dice within siblings; alternate whenever ratio worsens



<https://vega.github.io/vega/examples/treemap/>

70

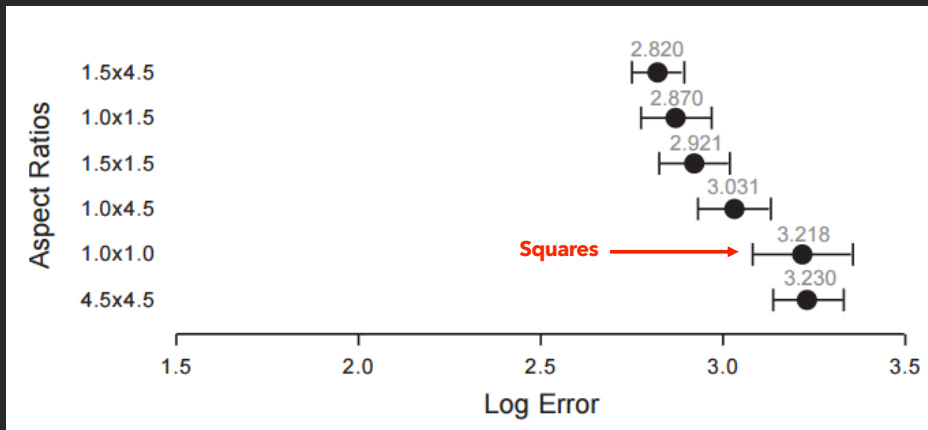
# Why Squares

## Posited Benefits of 1:1 Aspect Ratios

1. Minimize perimeter, reducing border ink.
2. Easier to select with a mouse cursor.  
*Validated by empirical research & Fitt's Law!*
3. Similar aspect ratios are easier to compare.  
*Seems intuitive, but is this true?*

71

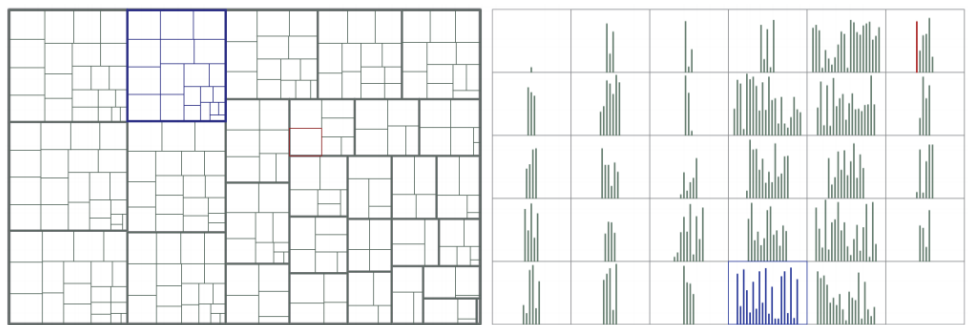
## Error vs. Aspect Ratio [Kong 10]



1. Comparison of squares has higher error!
2. Squarify works because it fails to meet its objective?

72

## Treemaps vs. Bar Charts [Kong 10]

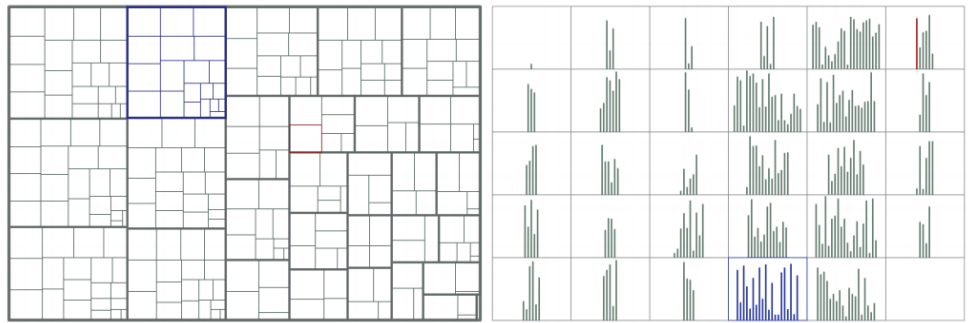


**Height more perceptually effective than area**

- What if element count is high?
- What about comparing groups of elements such as leaf values to internal node values?

74

## Treemaps vs. Bar Charts [Kong 10]



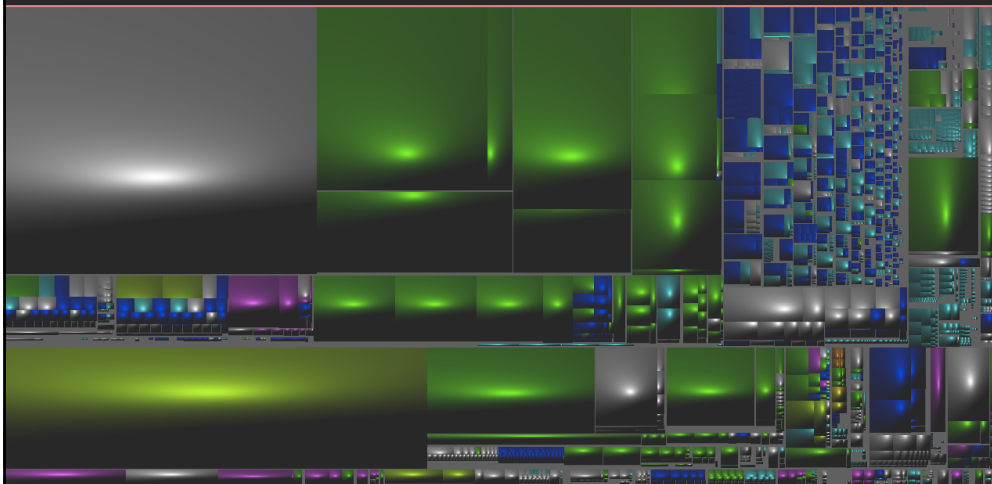
At low densities (< 4k elements), bar charts more accurate than treemaps for leaf-node comparisons.

At higher density, treemaps led to faster judgments.

Treemaps better for group-level comparisons.

75

## Cushion Treemaps [van Wijk 99]



Use shading to emphasize hierarchical structure

76

## Cascaded Treemaps [Lü 08]



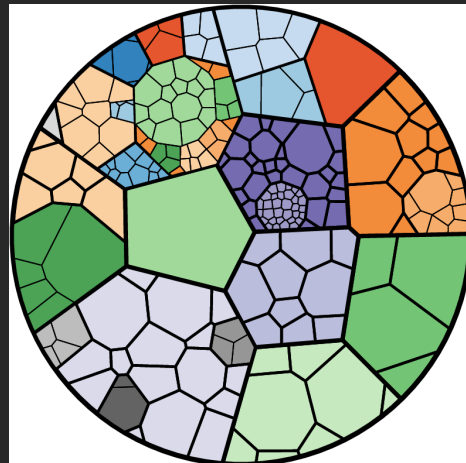
Use 2.5D effect emphasize hierarchical structure

77

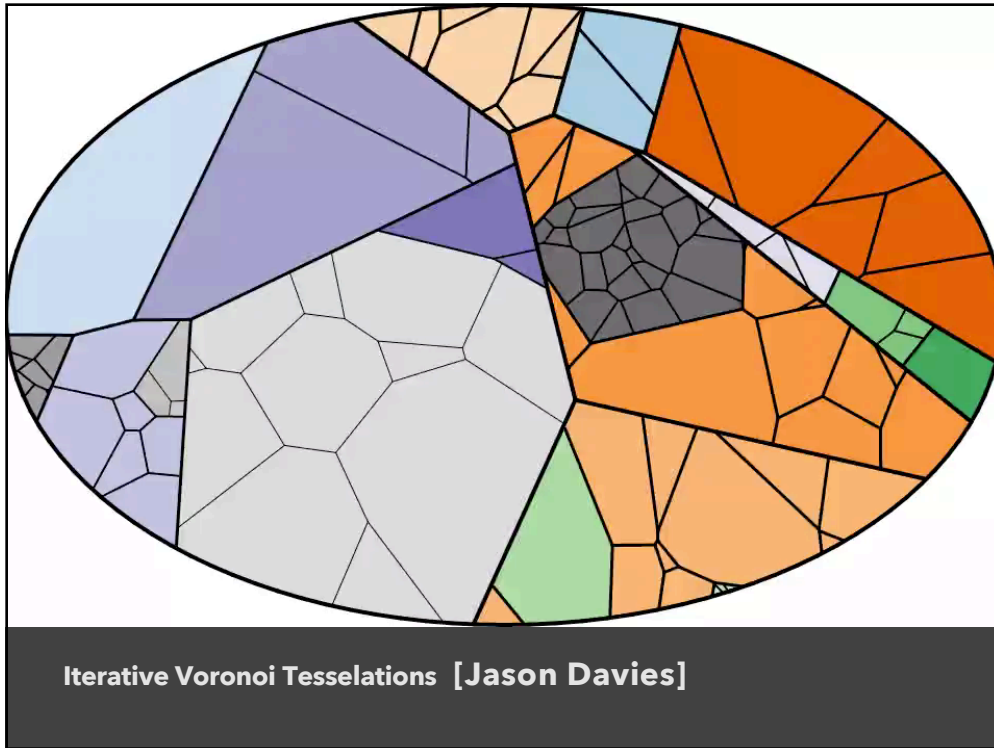
## Voronoi Treemaps [Balzer 05]

Treemaps with arbitrary polygonal shape and boundary

Uses iterative, eighted Voronoi tessellations to achieve cells with value-proportional areas



78



79

## Layered Diagrams

Signify tree structure using  
Layering  
Adjacency  
Alignment

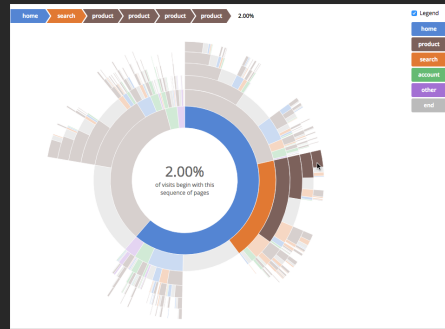
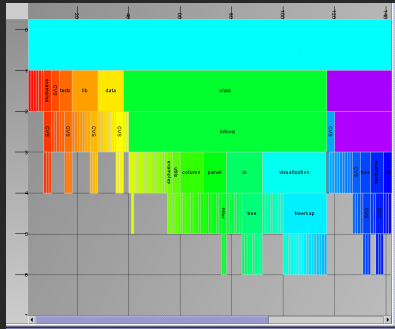


Involves recursive sub-division of space  
Can apply the same set of approaches as in node-link layout

80



# Icicle and Sunburst Trees



Higher-level nodes get a larger layer area, whether that is horizontal or angular extent  
 Child levels are layered, constrained to parent's extent

81

# Layered Tree Drawing

		Coffee			Espresso			
		Amaretto	Columbian	Decaf Irish Cr..	Caffe Latte	Caffe Mocha	Decaf Espresso	Regular Espre..
Central	Colorado	█	█	█		█	█	
	Illinois		█	█		█	█	
	Iowa		█	█		█	█	
	Missouri		█	█		█	█	
	Wisconsin		█	█		█	█	
East	Connecticut		█	█		█	█	
	Florida		█	█		█	█	
	Massachusetts		█	█		█	█	█
	New Hamps..	█	█	█		█	█	
	New York		█	█		█	█	█
South	Louisiana		█	█		█	█	
	New Mexico		█	█		█	█	
	Oklahoma		█	█	█	█	█	
	Texas		█	█		█	█	
West	California	█	█	█	█	█	█	
	Nevada		█	█		█	█	
	Oregon		█	█		█	█	
	Utah		█	█		█	█	
	Washington		█	█		█	█	
		-20K 0K 20K	-20K 0K 20K	-20K 0K 20K	-20K 0K 20K	-20K 0K 20K	-20K 0K 20K	-20K 0K 20K
		SUM(Profit)	SUM(Profit)	SUM(Profit)	SUM(Profit)	SUM(Profit)	SUM(Profit)	SUM(Profit)

82

# Node-Link Graph Layout

84

## Spanning Tree Layout

---

**Many graphs are tree-like or have useful spanning trees**

Websites, Social Networks

**Use tree layout on spanning tree of graph**

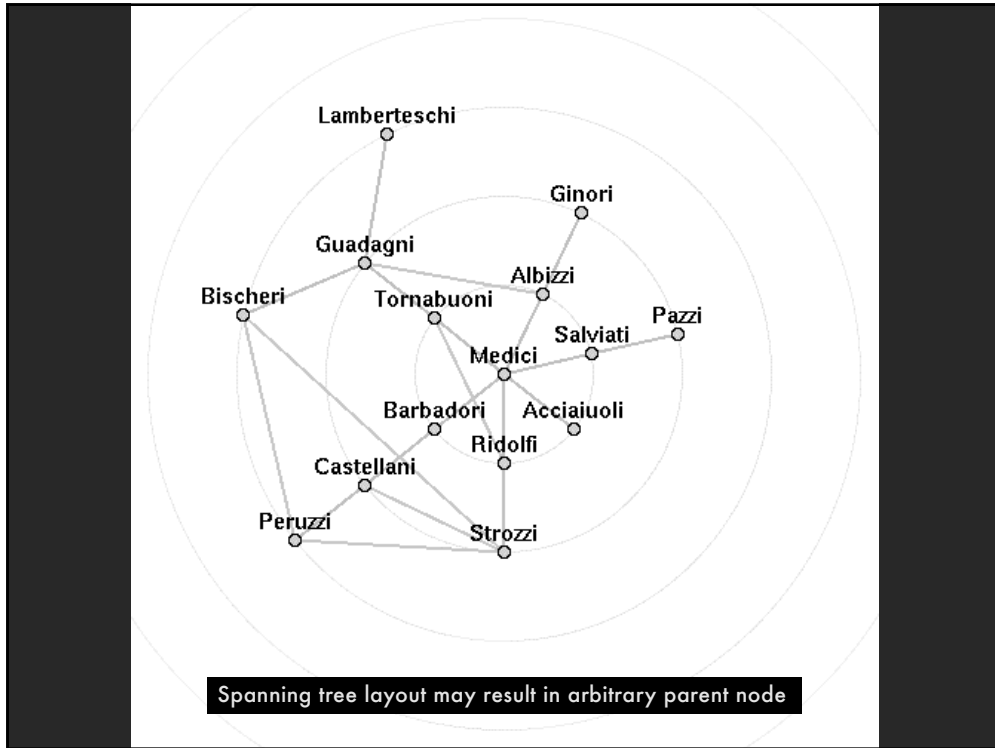
Trees created by BFS / DFS

Min/max spanning trees

**Fast tree layouts allow graph layouts to be recalculated at interactive rates**

**Heuristics may further improve layout**

86

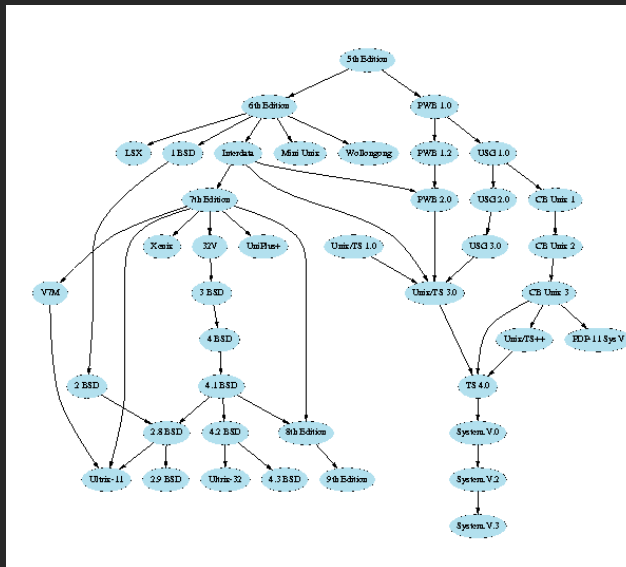


87

## Sugiyama-style graph layout

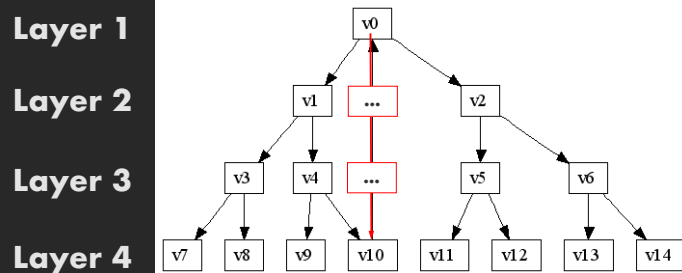
Evolution of the UNIX operating system

Hierarchical layering based on descent



88

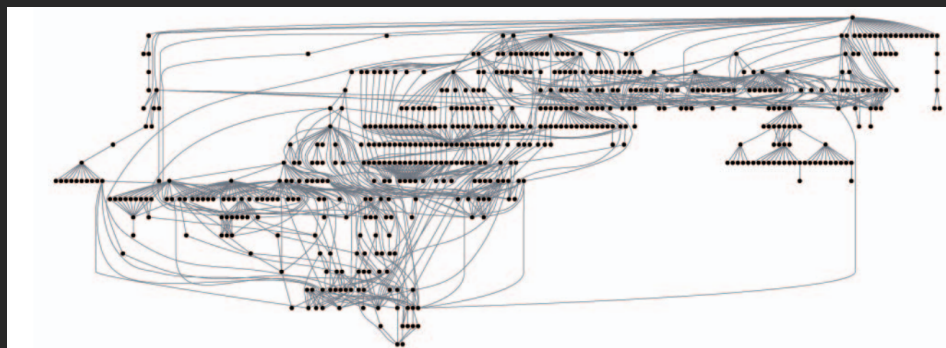
# Sugiyama-style graph layout



- Reverse some edges to remove cycles
- Assign nodes to hierarchy layers → Longest path layering
  - Create dummy nodes to “fill in” missing layers
- Arrange nodes within layer, minimize edge crossings
- Route edges – layout splines if needed

89

# Produces hierarchical layout



## Sugiyama-style layout emphasizes hierarchy

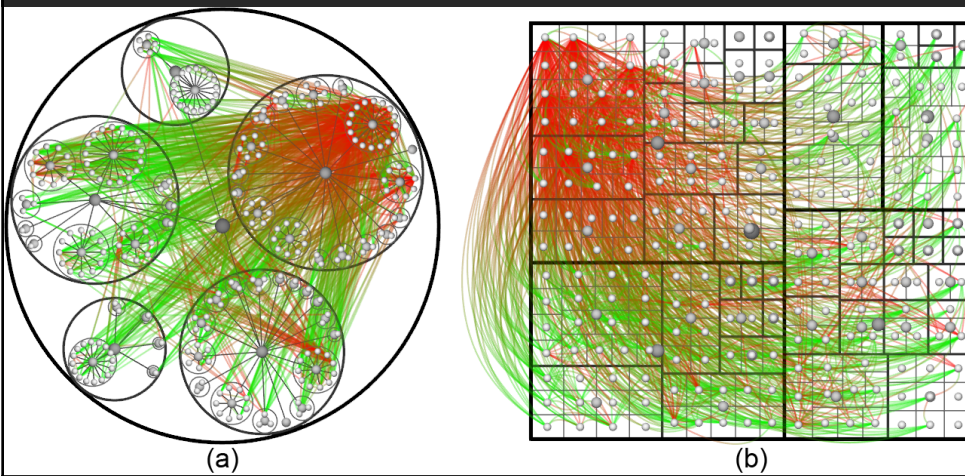
However, cycles in the graph may mislead.  
Long edges can impede perception of proximity.

90

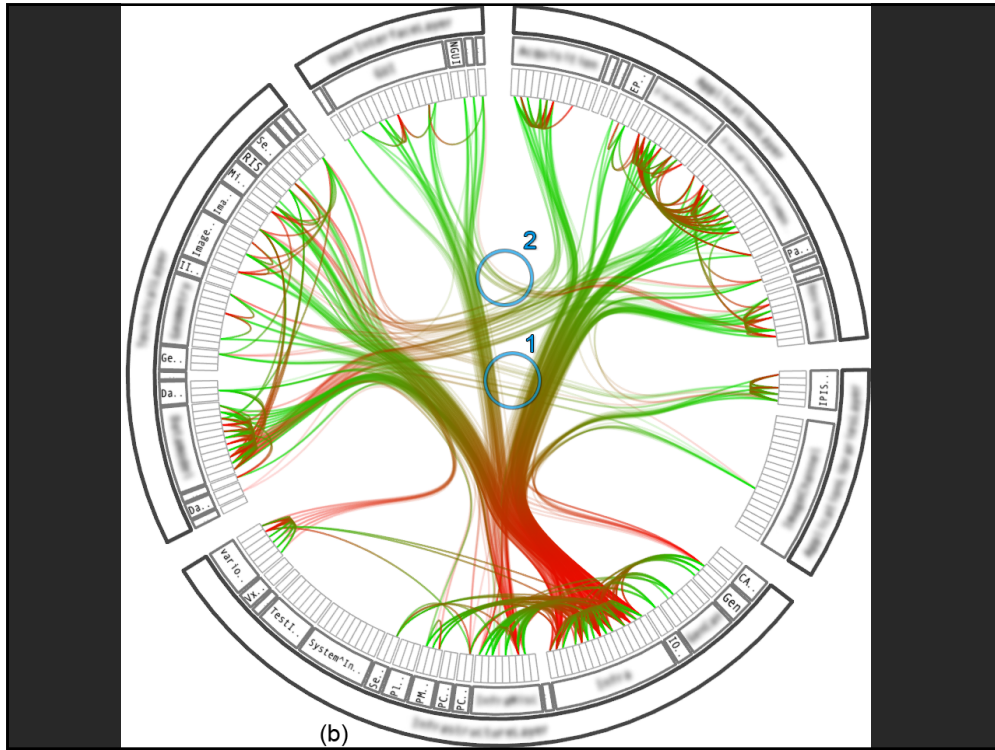
# Hierarchical Edge Bundles

92

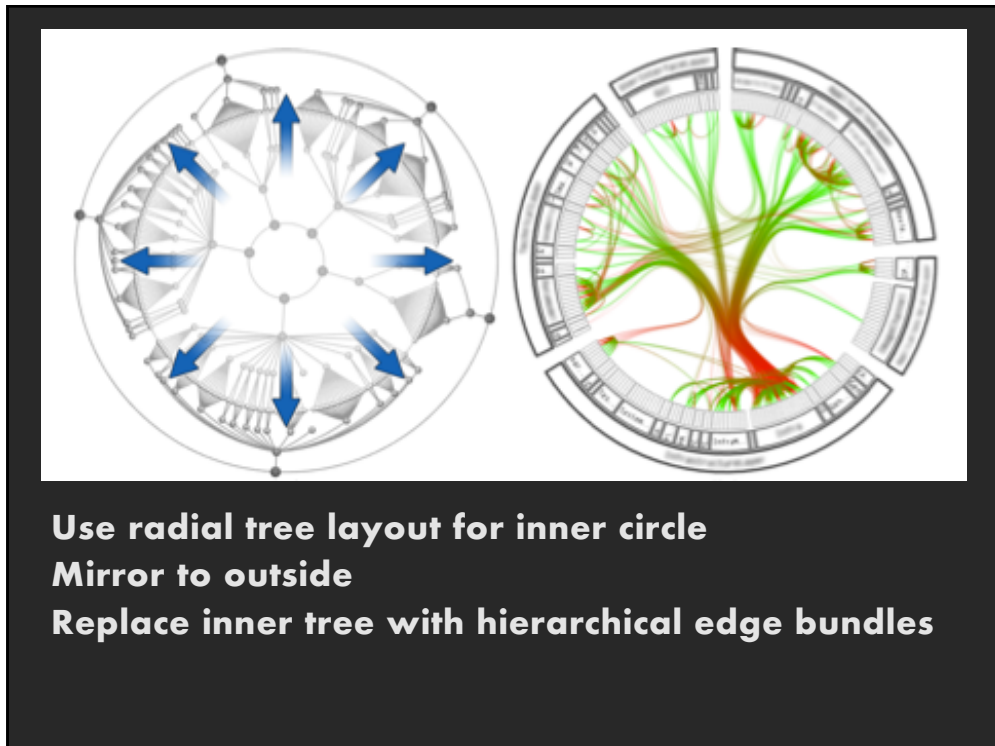
## Trees with Adjacency Relations



93

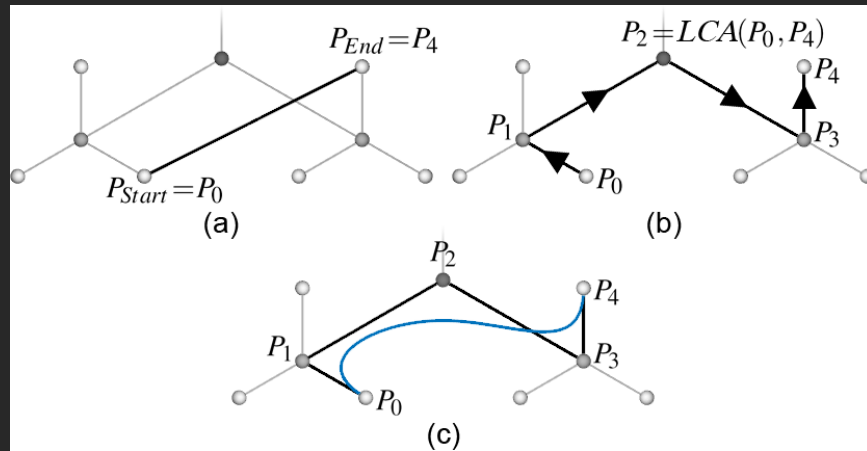


94



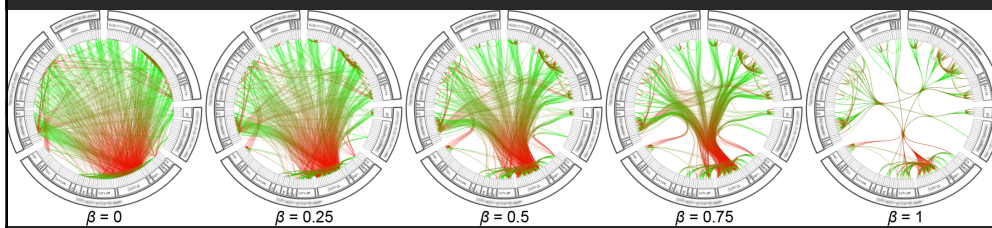
95

# Bundle Edges along Hierarchy



96

# Increasing Edge Tension



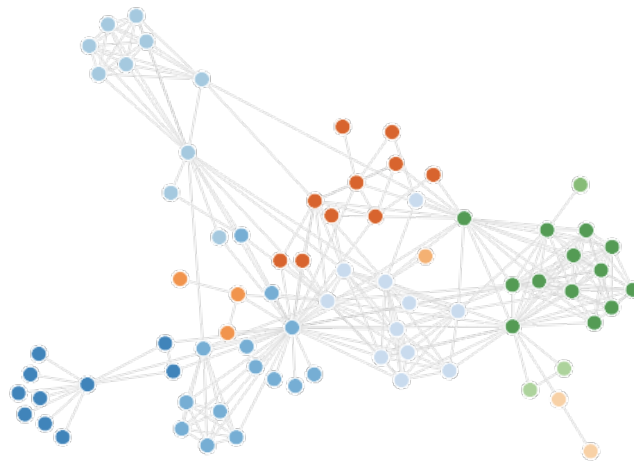
97





# Force-Directed Layout

100



**Interactive Example: Configurable Force Layout**

101

**Zephoria**

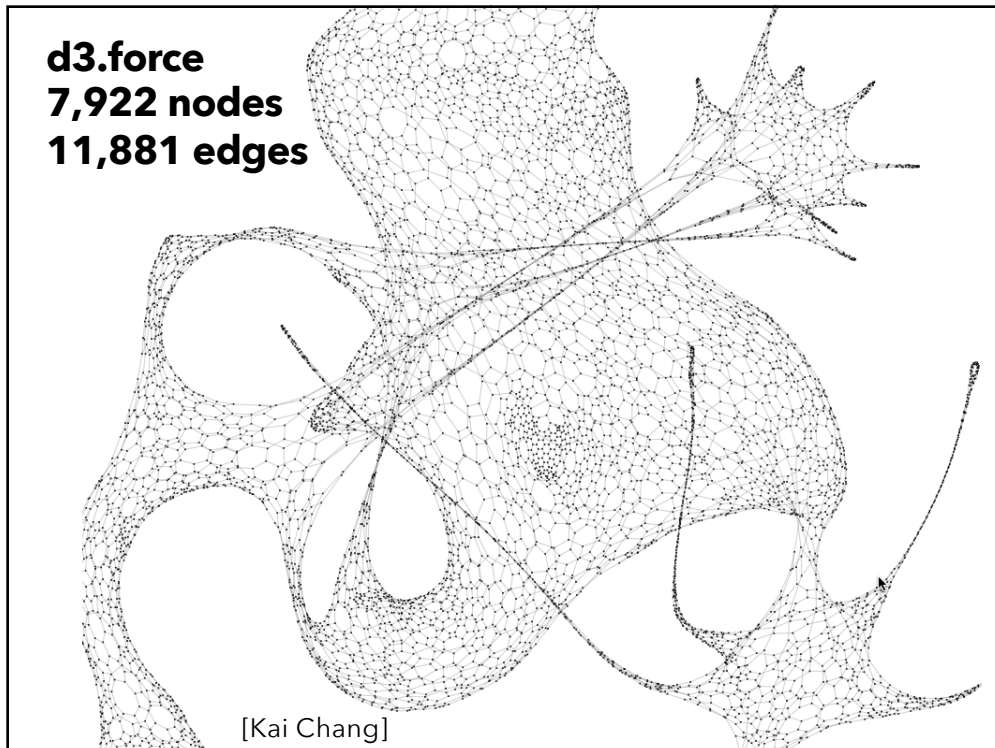
User ID 21721  
 Friends 266  
 Age ??  
 Gender  Female  
 Status  Single  
 Location San Francisco, CA  
 Hometown Lancaster, PA  
 Occupation researcher social networks, identity, context  
 Interests apophenia, observing people, culture, questioning power, reading, Buddhism, socialy, computer-mediated communication, social networks, technology, anthropology, stamping  
 Music Mushroom, Son Kite... iboga[Digital Structures], Ani Difranco, Downtempo, Theivery Corporation, Beth Orton, Morcheeba, Ween, White Stripes  
 Books Authors: Erving Goffman, Stanley Milgram, Jeanette Winterson, Eric Schlosser, Leslie Feinberg, Dorothy Allison, Italo Calvino, Hermann Hesse  
 TV Shows ?  
 Movies Koyaanisqatsi, Amelie, Waking Life, Tank Girl, The Matrix, Clockwork Orange, American Beauty, Fight Club, Boys Don't Cry  
 Member Since ?  
 Last Login 2003-10-21  
 Last Updated 2003-10-21  
 About [Some know me as danah...]  
 I'm a geek, an activist and an academic, fascinated by people and society. I see life as a very large playground and enjoy exploring its intricacies. I revel in life's chaos, while simultaneously providing my own insane element.  
 My musings: <http://www.zephoria.org/though.htm/>  
 Want to Meet Someone who makes life's complexities seem simply elegant.

102

# Use the Force!

<http://mbostock.github.io/d3/talk/20110921/>

103



104

## Force-Directed Layout

**Nodes = charged particles  
with air resistance**

$$F = q_i * q_j / d_{ij}^2$$

$$F = -b * v_i$$

**Edges = springs**

$$F = k * (L - d_{ij})$$

**D3's force layout uses velocity Verlet integration**

Assume uniform mass  $m$  and timestep  $\Delta t$ :

$$F = ma \rightarrow F = a \rightarrow F = \Delta v / \Delta t \rightarrow F = \Delta v$$

*Forces simplify to velocity offsets!*

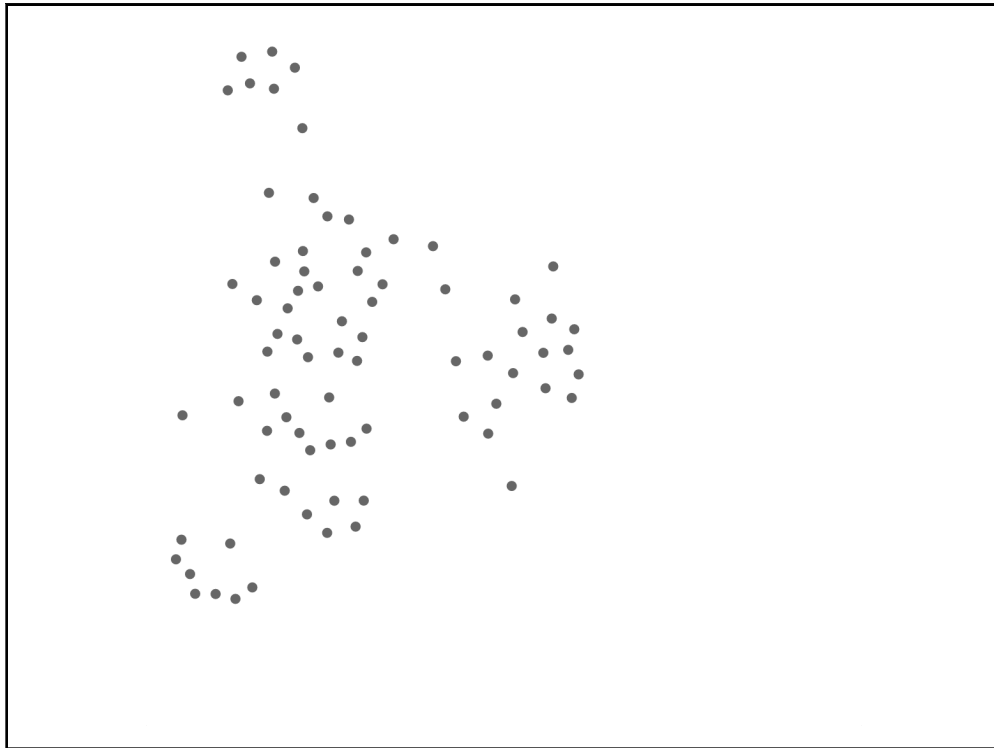
**Repeatedly calculate forces, update node positions**

Naïve approach  $O(N^2)$

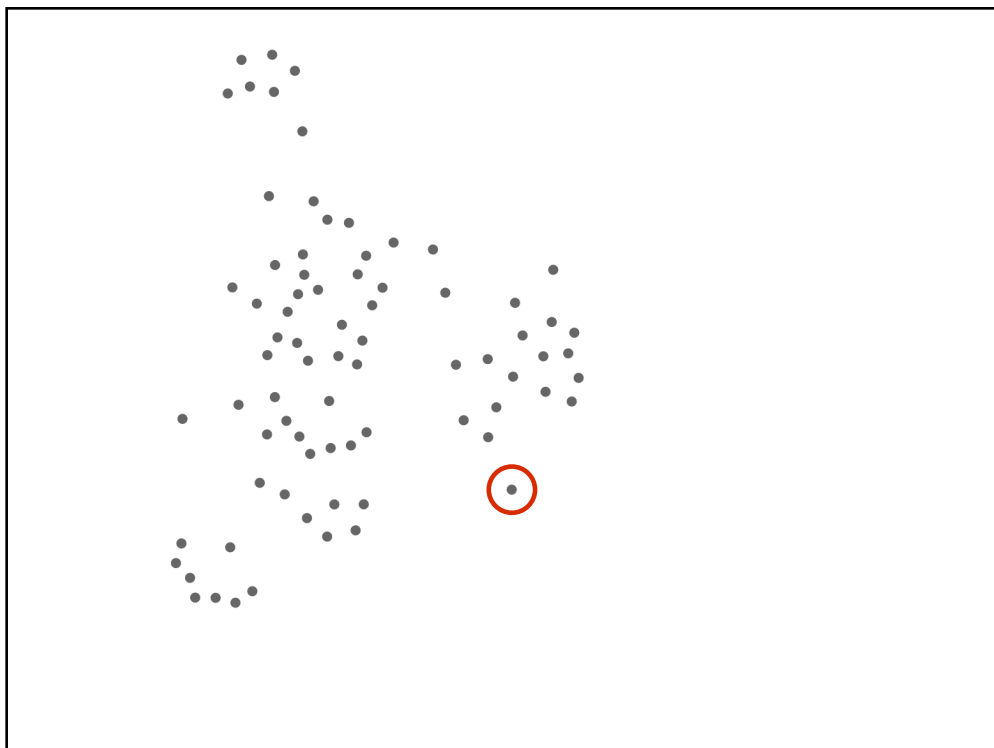
Speed up to  $O(N \log N)$  using quadtree or k-d tree

Numerical integration of forces at each time step

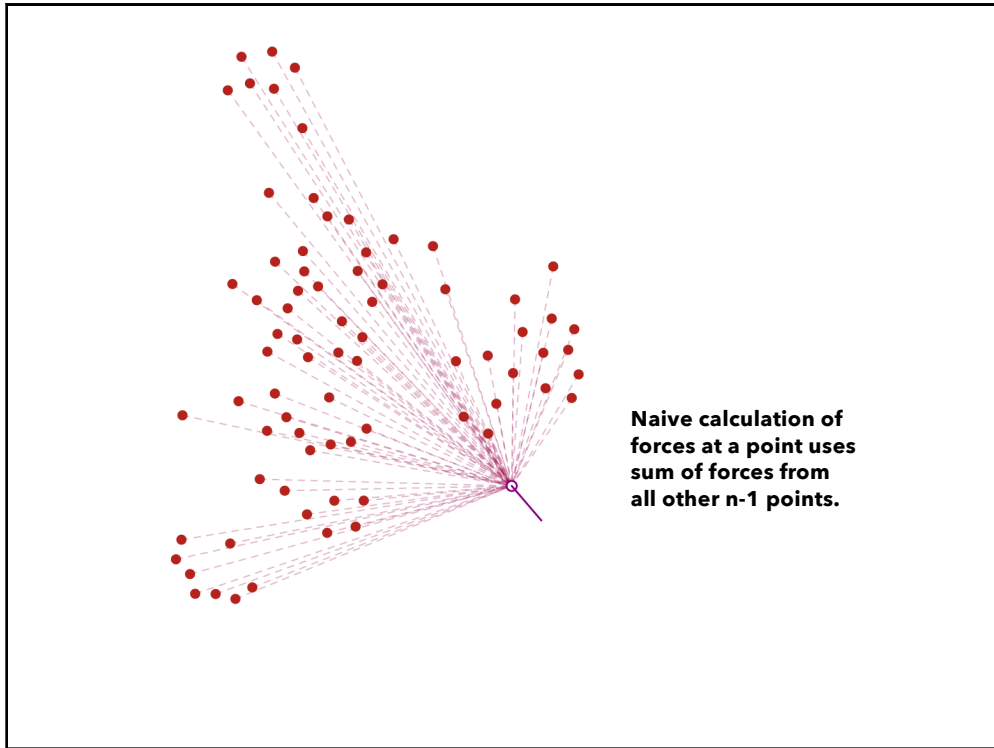
105



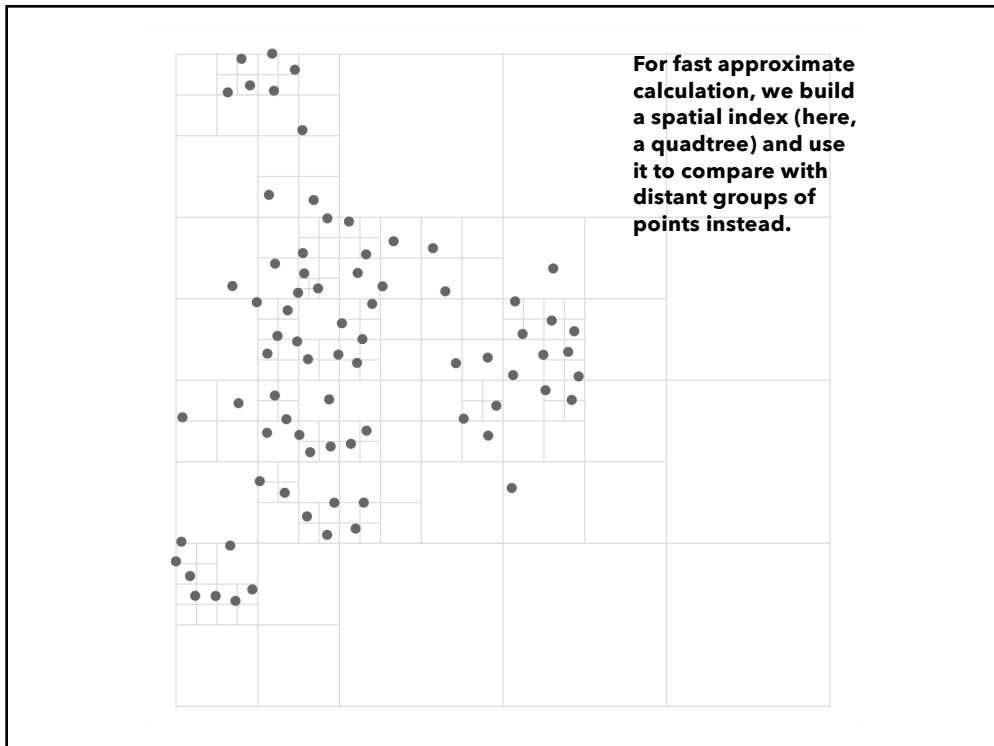
106



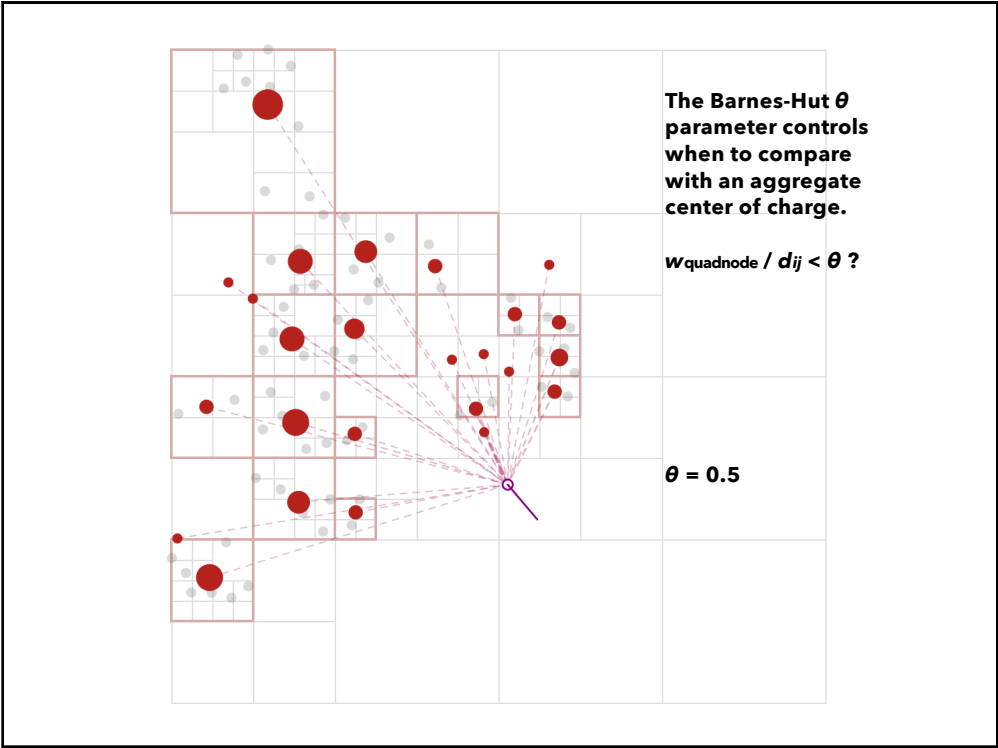
107



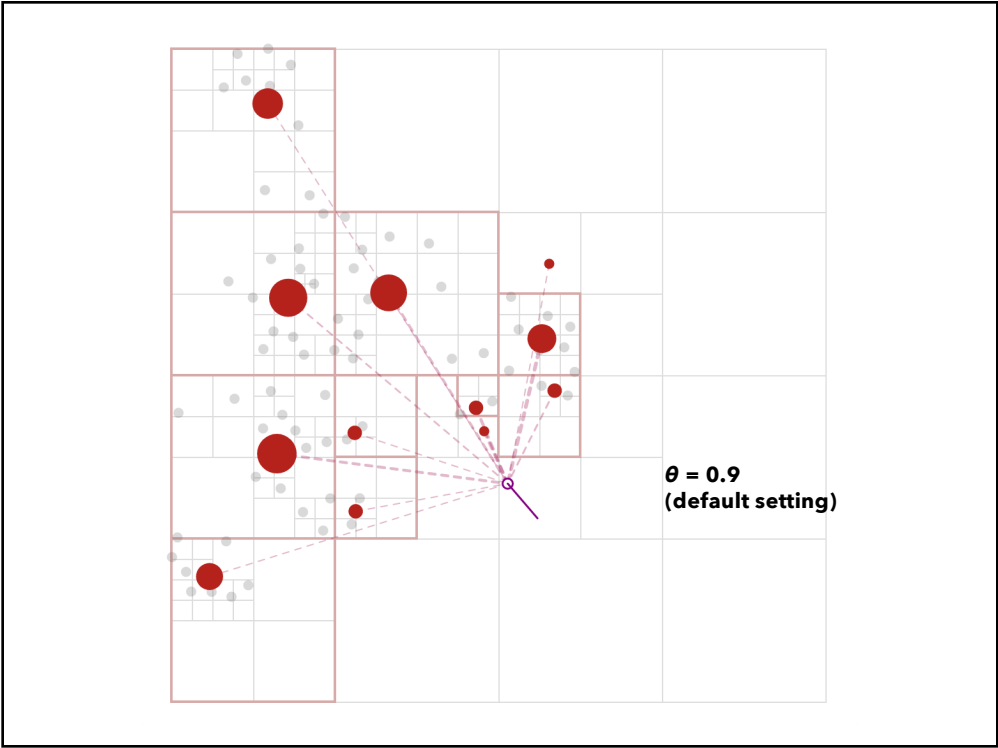
108



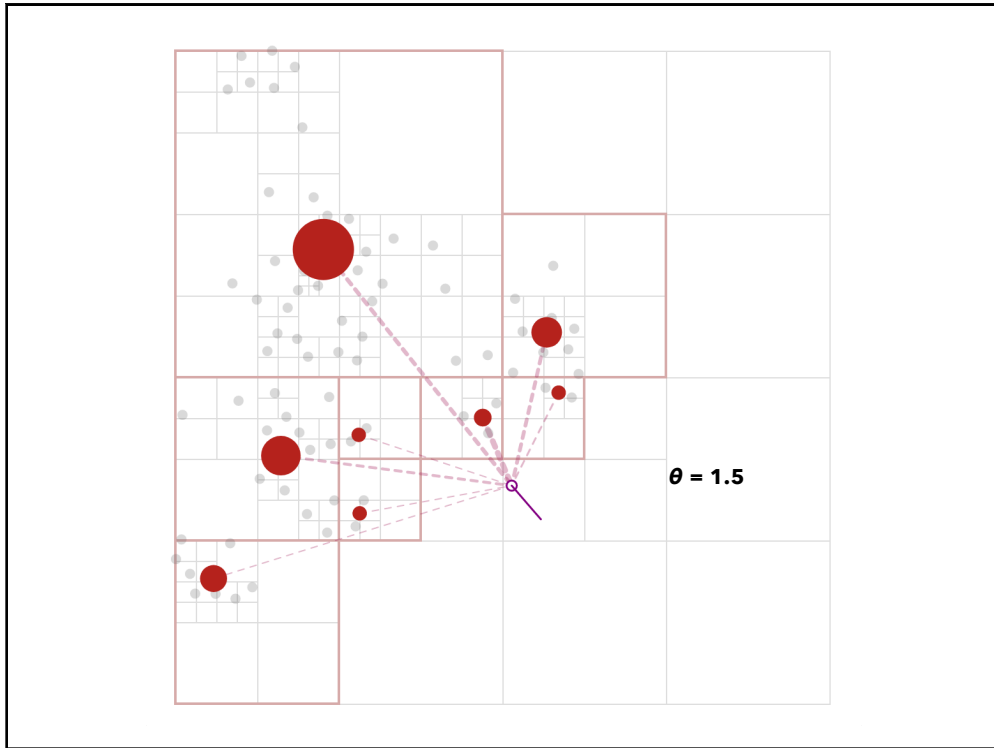
109



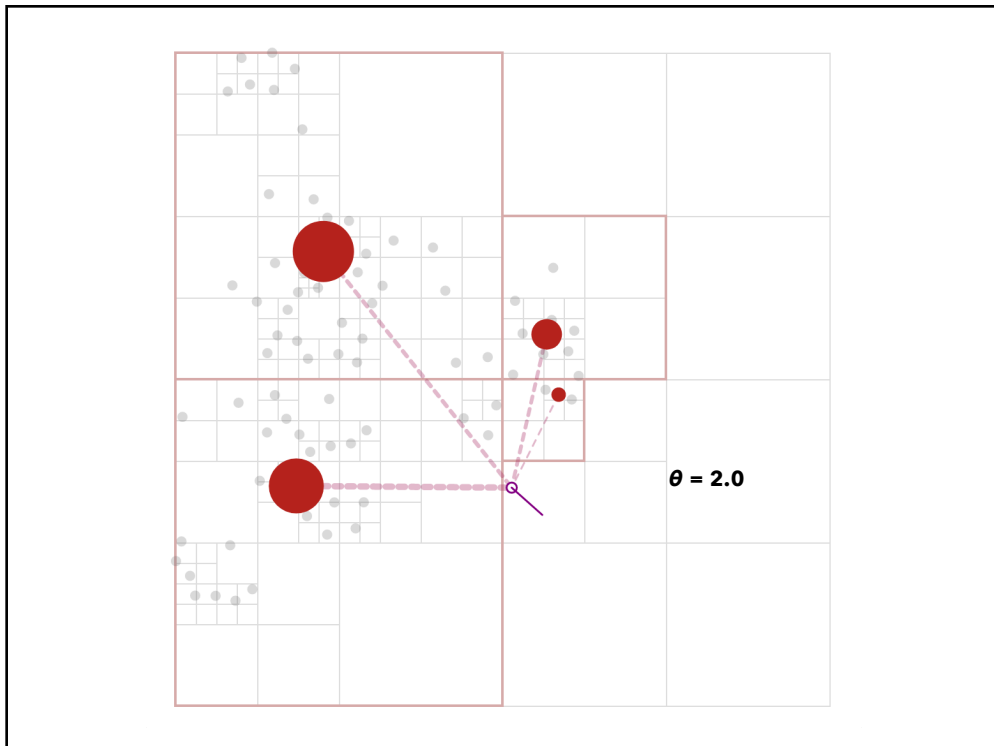
110



111



112



113