

# NETWORK ANALYSIS

CS 448B | Fall 2023

MANEESH AGRAWALA

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A graphic for SB Nation's 'Chart Party' series. On the left, a photograph shows a baseball pitcher in a grey uniform on the mound, a man in a white shirt and blue jeans standing on the infield, and a catcher in full gear crouching behind home plate. The background is a red wall with the text 'SB NATION CHART PARTY' in white and black. On the right, a dark grey vertical panel contains the text 'What if Barry Bonds had played baseball without a bat?' in white and orange.

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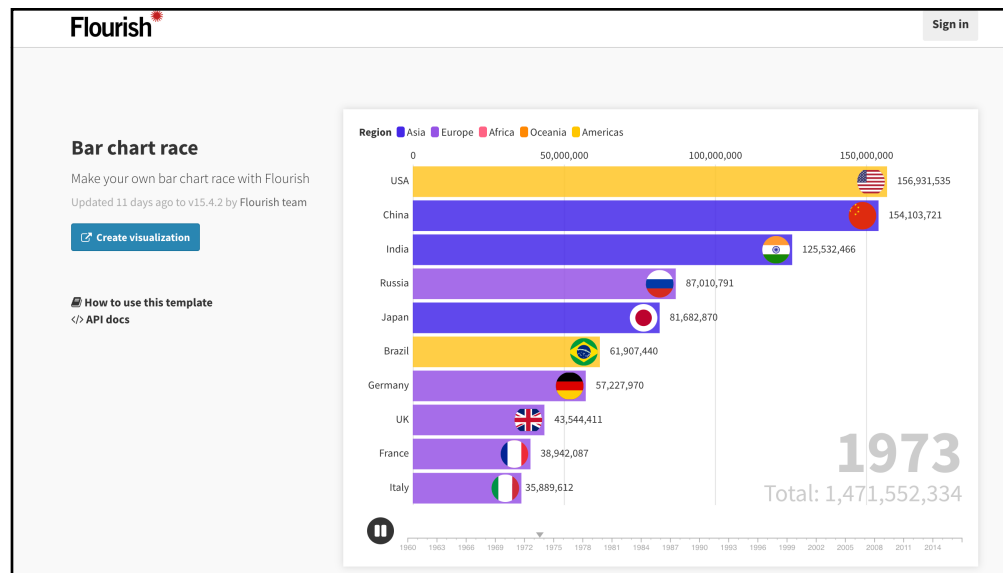
## READING RESPONSE: QUESTIONS/THOUGHTS

... When **considering visualizations as a form of art or a medium for storytelling, how important is explaining every single detail?** This issue of lack of clarity comes up quite frequently in class and I don't think I fully understand how it is a problem. When you look at a painting, you don't expect to get an explanation for artistic choices like brush strokes. I wonder why when conveying data that those choices seem to matter so much more.

**Given the drawbacks of animation** that we saw listed in lecture and the advice to use it very carefully because viewers cannot track many moving objects at once, **at what point should we prioritize engagement over clarity in a single viewing?** [...] My thought is that a **viewer can replay an animation as many times as they'd like** so that they can focus on different objects of interest, but engagement is something they cannot control.

...I discovered this article on policyviz: <https://policyviz.com/2019/08/06/observations-on-animation-in-data-visualization/>. **The article raises the point that animation is rarely necessary** and is often used poorly. However, they demonstrate an example of a **very effective use of animation in data visualization: bar chart races**. Bar chart races are popular videos on social media platforms such as Twitter that depict changes in trends over time as a race, using animation to depict both changes in trends, as well as to tell a story over time.

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## LAST TIME: NETWORK LAYOUT

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## NODE-LINK GRAPH VISUALIZATION

Nodes connected by lines/curves

**Sugiyama-Style Layout** - arranged by depth

**Force-Directed Layout** - physical simulation

**Attribute-Driven Layout** - arranged by value

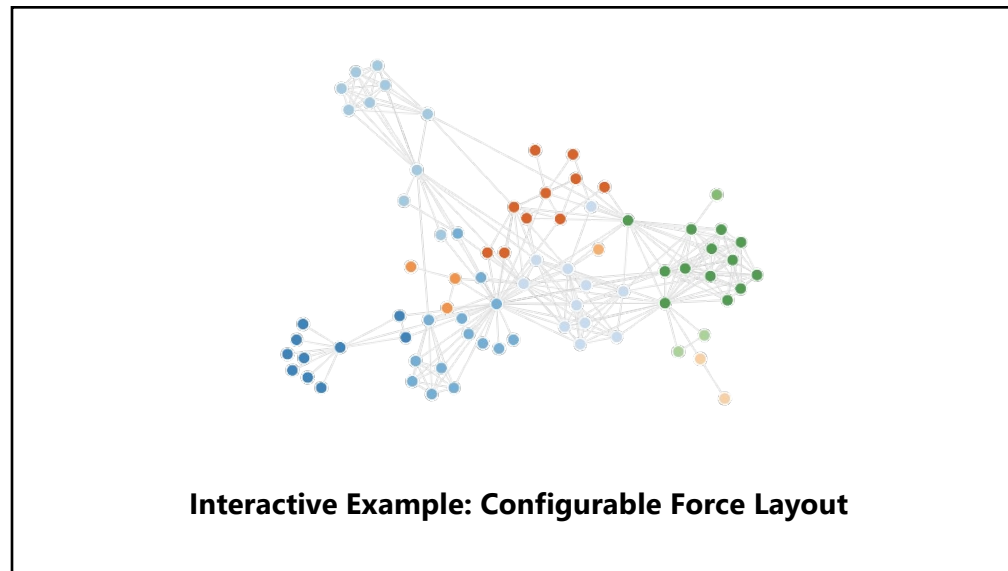
**Constraint-Based Layout** - optimization

**Arc Diagrams** - aligned layout

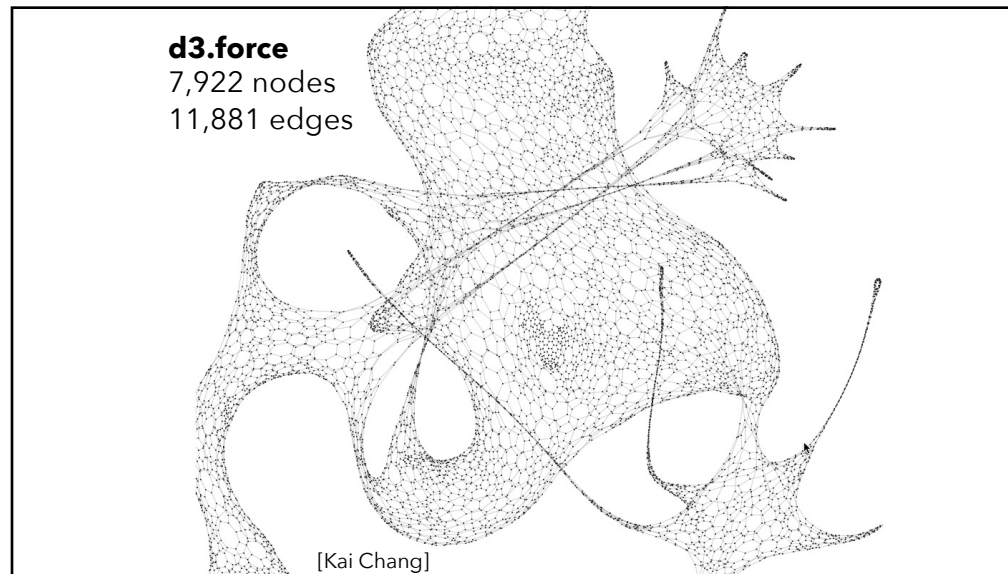
6

# FORCE-DIRECTED LAYOUT

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**Use the Force!**

<http://mbostock.github.io/d3/talk/20110921/>

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## LAYOUT BY PHYSICS SIMULATION

Nodes = charged particles  
with air resistance

$$F = q_i * q_j / d_{ij}^2$$

Edges = springs

$$F = -b * v_i$$

$$F = k * (L - d_{ij})$$

At each timestep, calculate forces acting on nodes.

Integrate for updated velocities and positions.

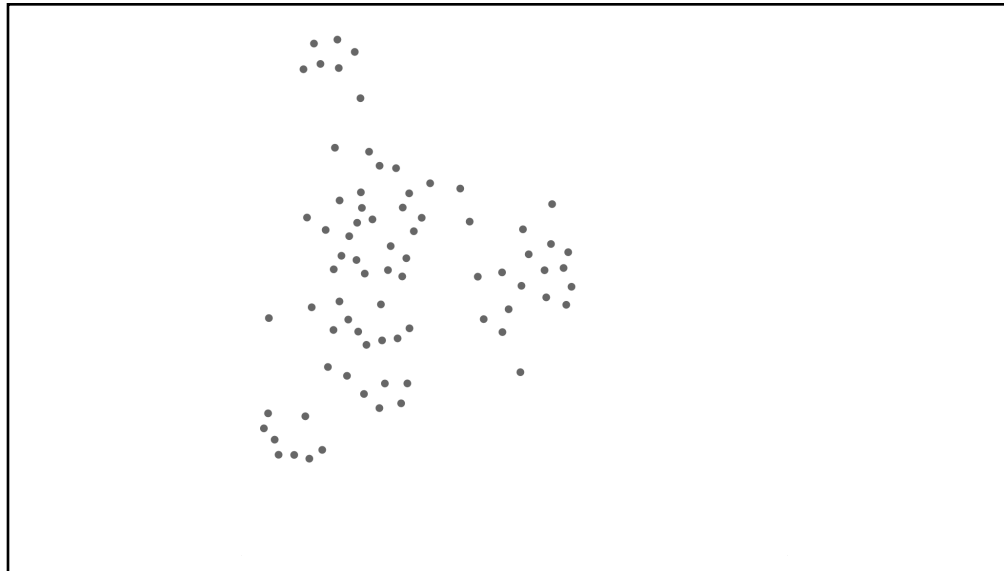
D3's force layout uses **velocity Verlet** integration

Assume uniform mass  $m$  and timestep  $\Delta t$ :

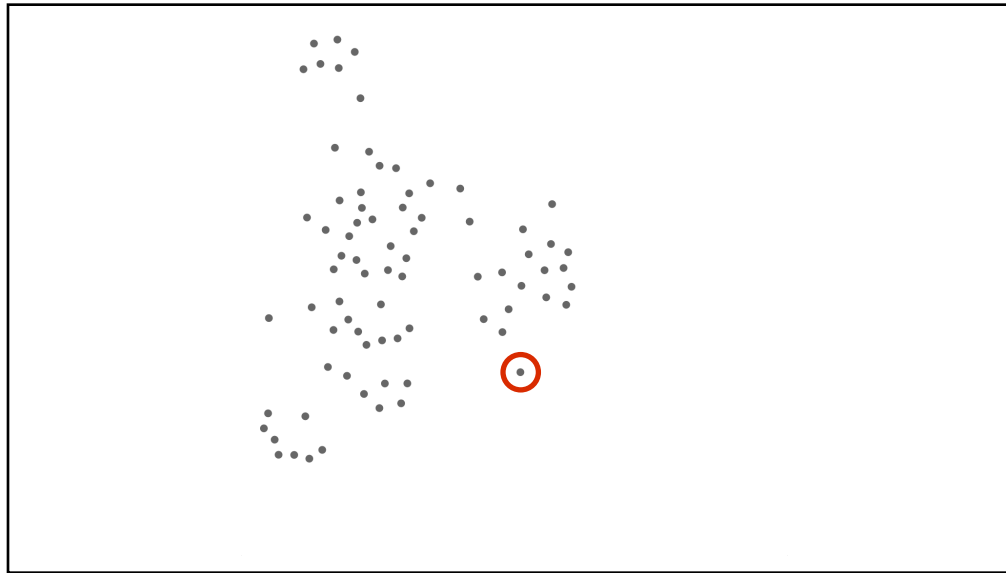
$$F = ma \rightarrow F = a \rightarrow F = \Delta v / \Delta t \rightarrow F = \Delta v$$

*Forces simplify to velocity offsets!*

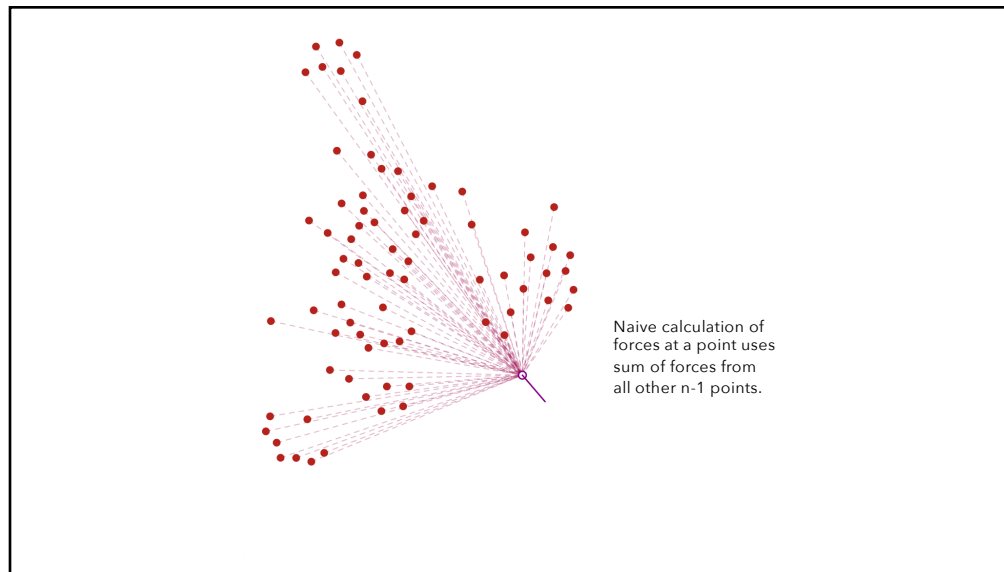
12



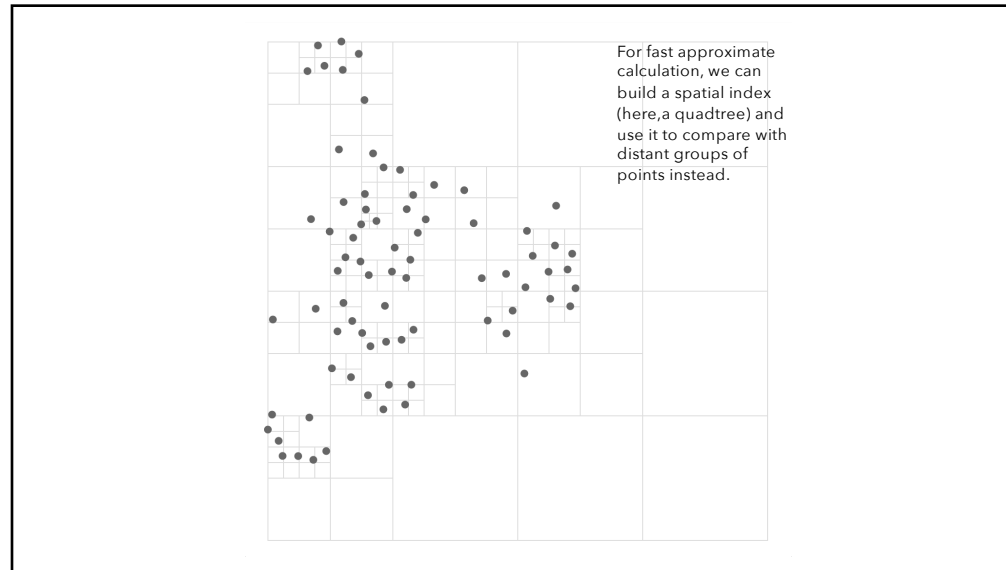
13



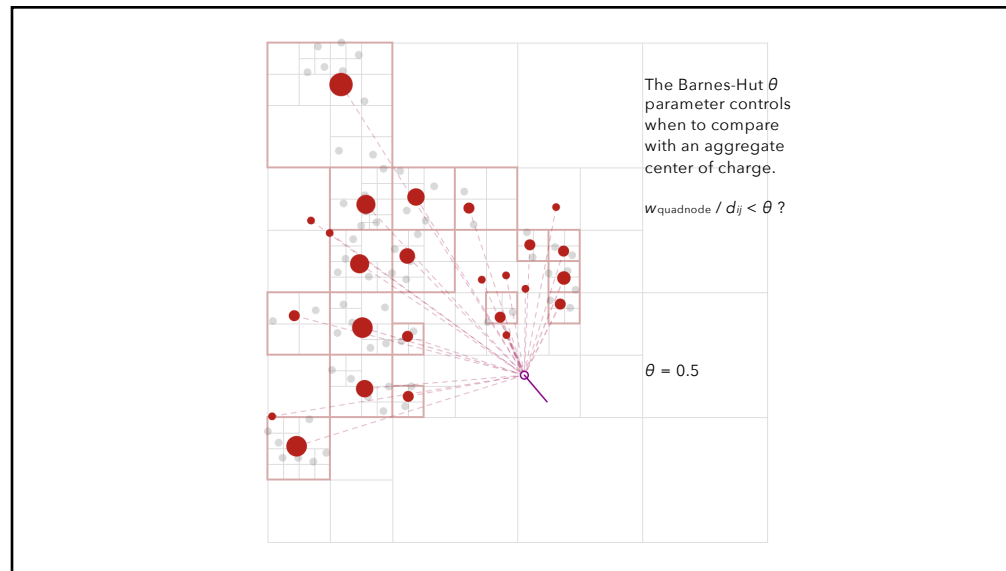
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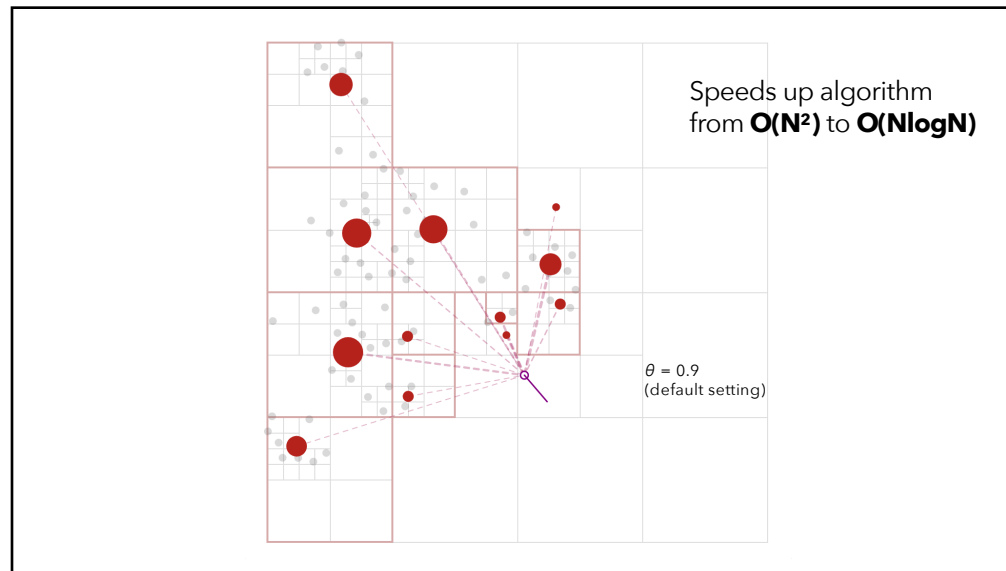


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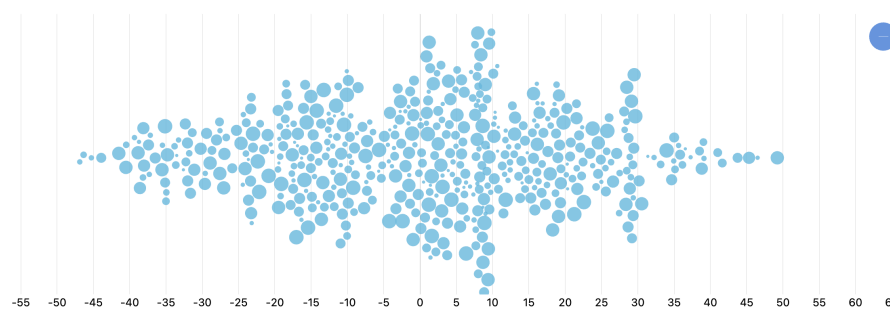
## CUSTOMIZED FORCE LAYOUTS <https://www.amcharts.com/demos/bee swarm/>

Different forces can be composed to create variety of custom layouts

A **beeswarm plot** can be made by combining:

Attractive **X** and **Y** forces to draw nodes of a certain category to a desired point

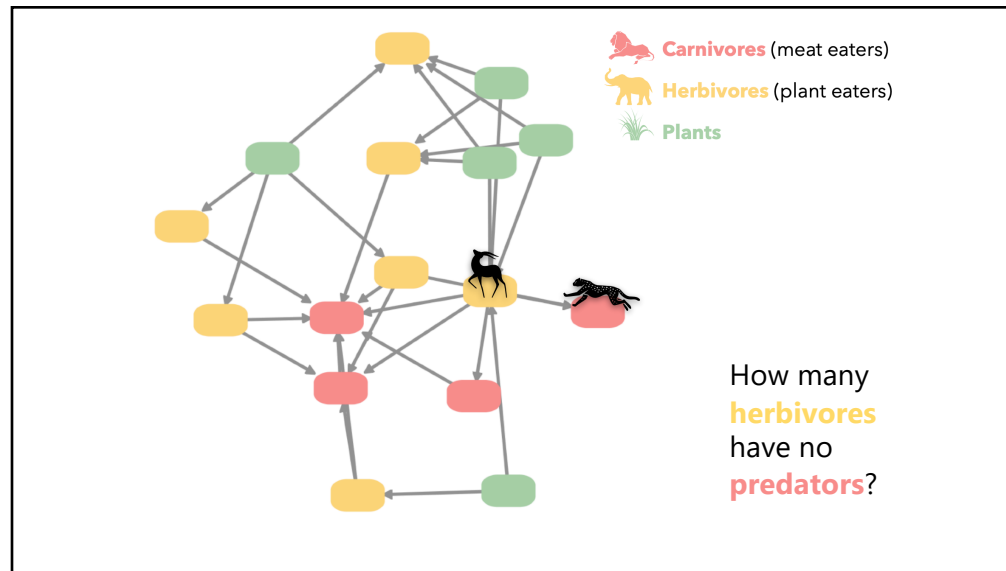
**Collide** force to detect collision & remove overlap



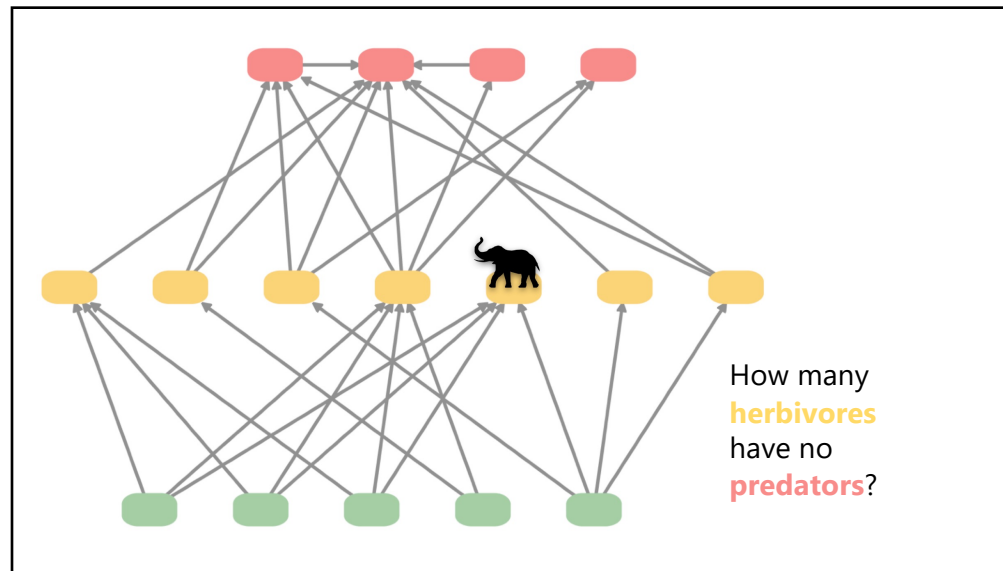
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## ATTRIBUTE-DRIVEN LAYOUT

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## ATTRIBUTE-DRIVEN LAYOUT

Large node-link diagrams **get messy!**

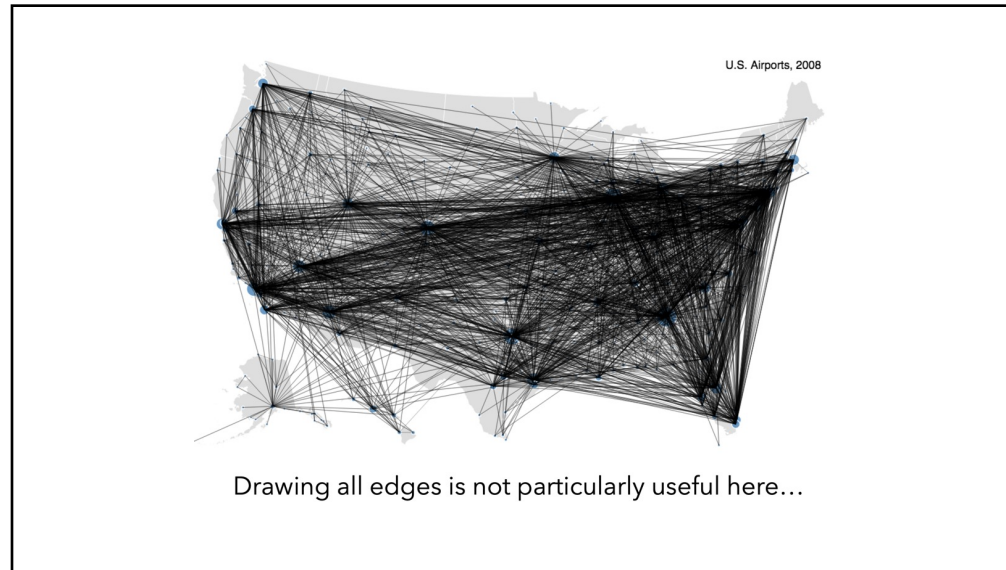
Can we exploit additional structure?

*Idea:* Use **data fields/attributes** associated with nodes or edges to perform layout (e.g., scatter plot based on node values)

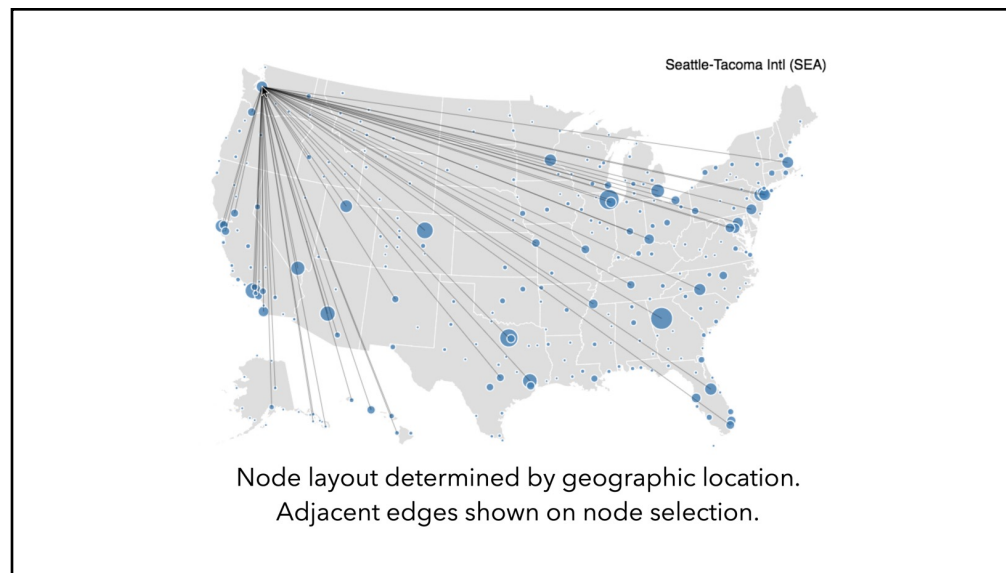
Attributes may also be statistical properties of the graph

Can apply dynamic queries & brushing on attributes/fields to explore...

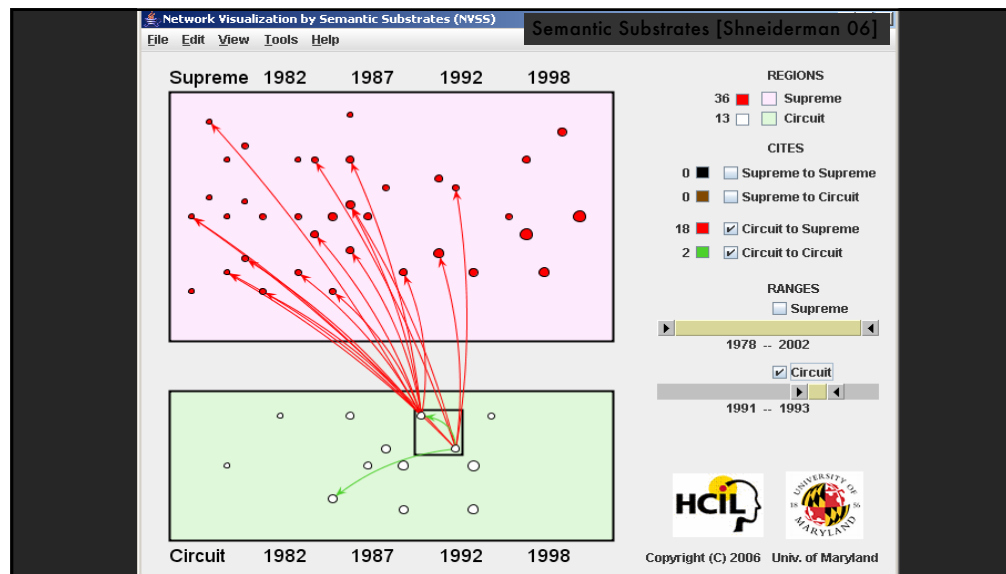
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**CONSTRAINT-BASED LAYOUT**

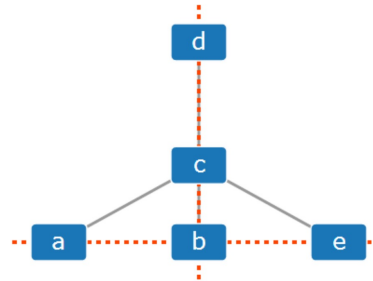
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## CONSTRAINT-BASED LAYOUT

### Treat layout as an optimization problem

Define layout using an *energy model* along with *constraint equations* the layout should obey

Use optimization algorithms to solve:



### Position Constraints

a must be to the **left** of b

d, c, and b must have the same **x position**

a, b, and e must have the same **y position**

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## OPTIMIZING AESTHETICS

Minimize edge crossings

Minimize area

Minimize line bends

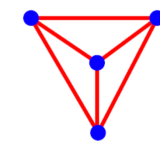
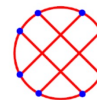
Minimize line slopes

Maximize smallest angle between edges

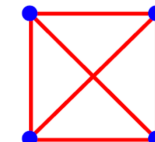
Maximize symmetry

### but, can't do it all

Optimizing these criteria is often NP-Hard, and requires approximations



min # crossings

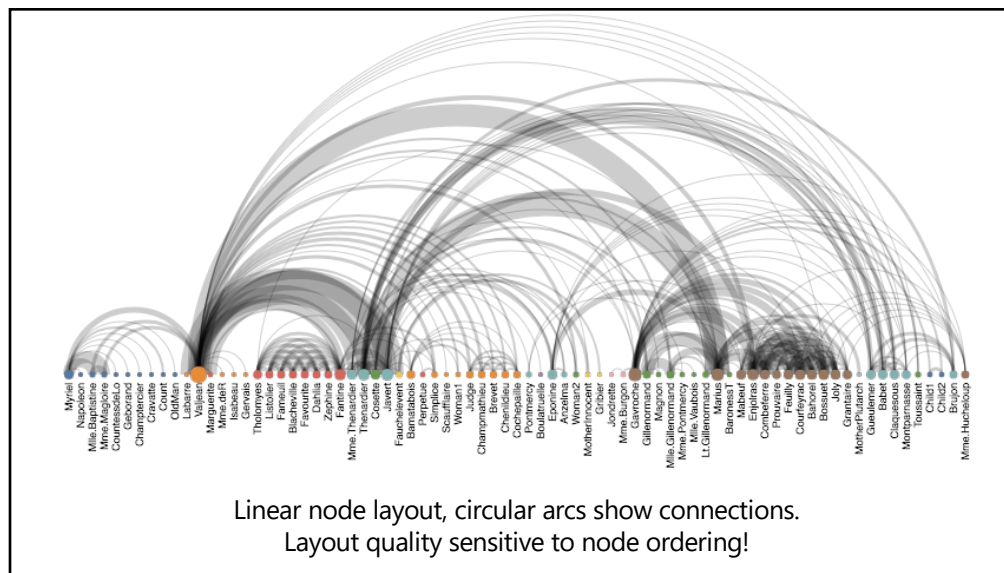


max symmetries

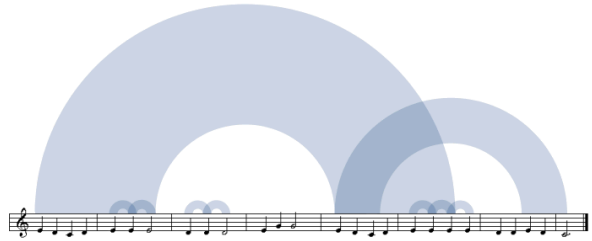
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# ARC DIAGRAMS


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For example, the picture above was built from the first line of a very simple piece: *Mary Had a Little Lamb*. Each arch connects two identical passages. To clarify the connection between the visualization and the song, in this diagram the score is displayed beneath the arches.



This diagram visualizes the refrain from the folk song *Clementine*. As you would expect, the refrain consists of multiple repetitions of the same passage--and that is exactly what the diagram shows. The score isn't shown in this diagram since the notes would be too small to read.

**The Shape of Song**  
[Wattenberg 2001]

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## NODE-LINK GRAPH VISUALIZATION

**Sugiyama-Style Layout** - arranged by depth

**Force-Directed Layout** - physical simulation

**Attribute-Driven Layout** - arranged by value

**Constraint-Based Layout** - optimization

**Arc Diagrams** - aligned layout

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## NODE-LINK GRAPH VISUALIZATION

**Sugiyama-Style Layout** - arranged by depth

**Good:** Structure-based analysis of hierarchical relationships

**Bad:** Browsing and path following due to long edges

**Force-Directed Layout** - physical simulation

**Good:** Structure-based analysis of closely related elements

**Bad:** Browsing and summarization of dense networks

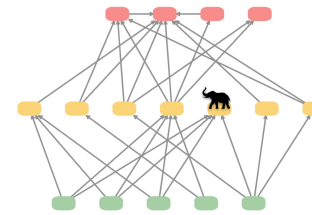
**Attribute-Driven Layout** - arranged by value

**Good:** Enables attribute-based analysis tasks

**Bad:** Difficult to design layouts appropriate to revealing attributes and network structure

**Constraint-Based Layout** – optimization

**Arc Diagrams** - aligned layout



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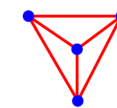
**Bad:** Difficult to design layouts appropriate to revealing attributes and network structure

**Constraint-Based Layout** – optimization

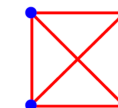
**Good:** Graph layout based on structural/aesthetic properties

**Bad:** Difficult to select appropriate constraints

**Arc Diagrams** - aligned layout



min # crossings



max symmetries

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## NODE-LINK GRAPH VISUALIZATION

### Sugiyama-Style Layout - arranged by depth

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### Constraint-Based Layout – optimization

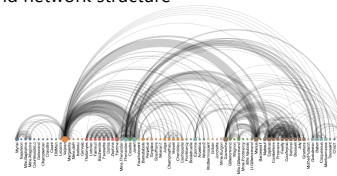
**Good:** Graph layout based on structural/aesthetic properties

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### Arc Diagrams - aligned layout

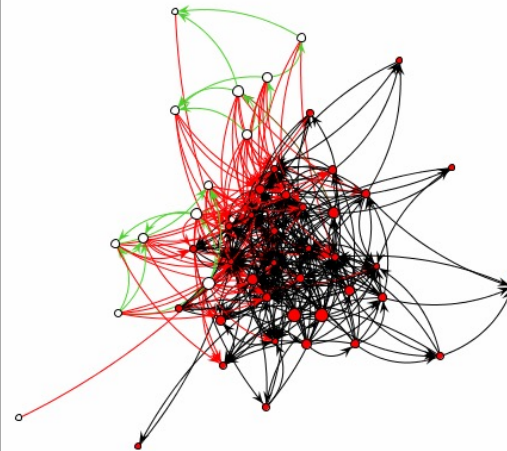
**Good:** Summarization and comparison of overall structure

**Bad:** Order matters for node layout; Structure-based and path following



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## LIMITATIONS OF NODE-LINK LAYOUTS



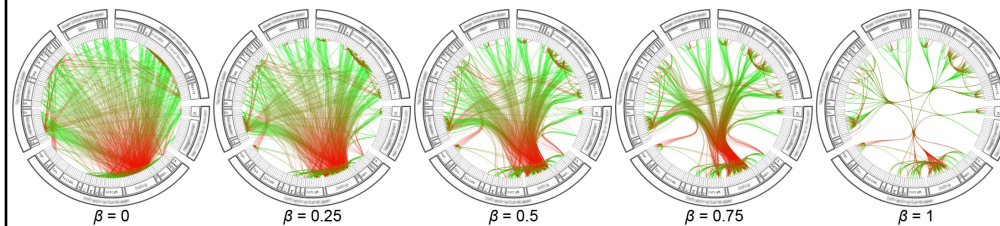
Edge crossings and occlusions!  
Poor scalability...

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## HIERARCHICAL EDGE BUNDLING

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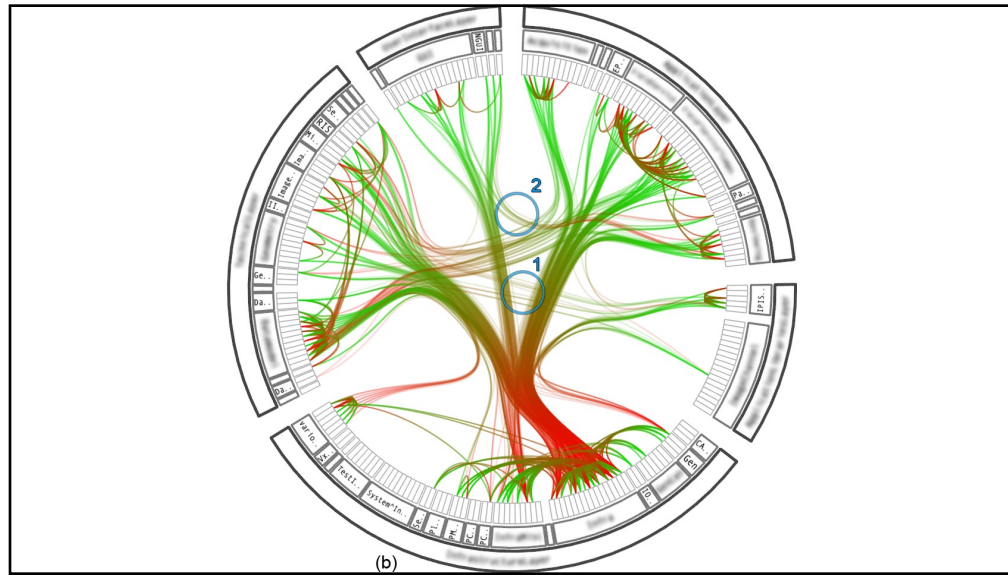
## HIERARCHICAL EDGE BUNDLING



Given a tree with additional *adjacency* edges (usually between leaves)

Bundle edges with varying amounts of tension – helping to reveal common connections between subtrees

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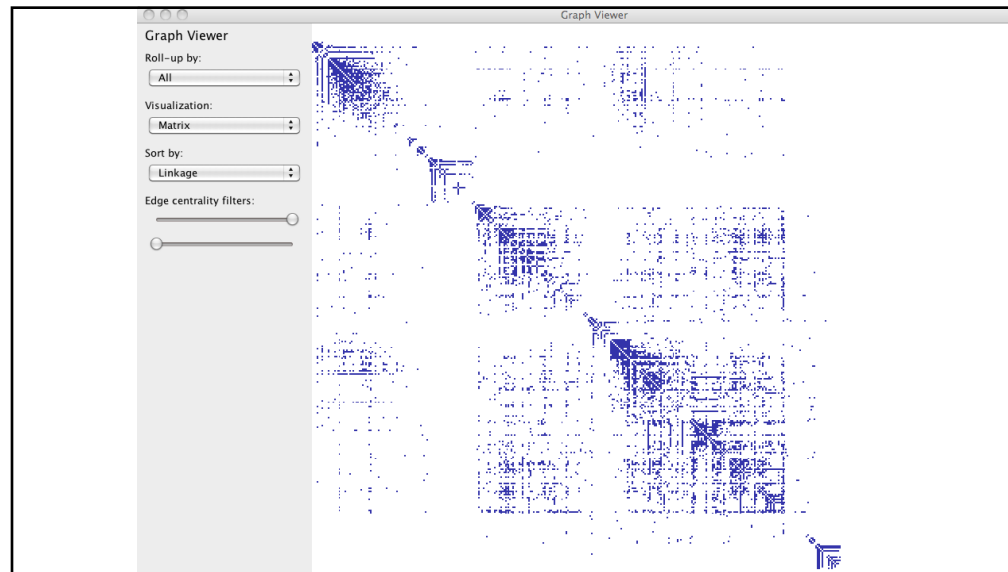


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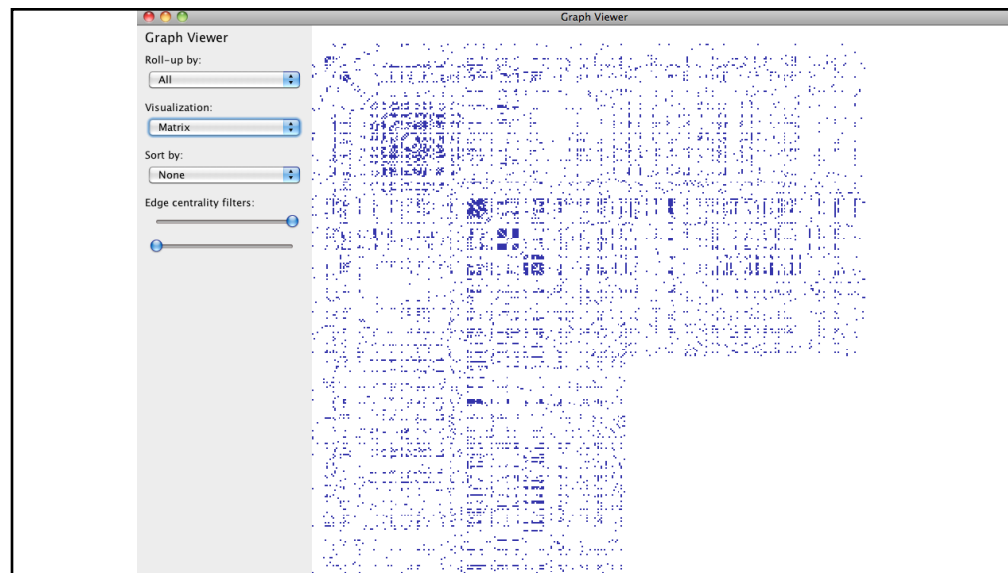
# MATRIX DIAGRAMS

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## **SUMMARY: TREES AND NETWORKS**

### **Tree Layout**

Indented / Node-Link / Enclosure / Layers  
Focus+Context techniques for scale

### **Graph Layout**

Sugiyama Layout  
Force-Directed Layout  
Attribute-Driven Layout  
Constraint Layout  
Arc Diagrams  
Matrix Diagrams

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## **ANNOUNCEMENTS**

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# FINAL PROJECT

## Design Review Nov 27 and 29

### Data analysis/explainer

Analyze dataset in depth & make a visual explainer

### Deliverables

An article with multiple different interactive visualizations

Short video (2 min) demoing and explaining the project

### Schedule

Project proposal: Mon 11/6

Design Review and Feedback: 9<sup>th</sup> week of quarter, 11/27 and 11/29

Final code and video: Sun 12/10 8pm

### Grading

Groups of up to 3 people, graded individually

Clearly report responsibilities of each member

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# FINAL PROJECT GUIDELINES

## Consider the audience

Your visual explainer should be of interest to a group of people beyond your immediate circle (an explainer about your own Spotify data unlikely be of interest to others you don't know)

## Pick relatively less explored topics/datasets

Do some research on what has already been done for the topic/dataset(s)

Certain data like songs (e.g. Spotify) or movies (e.g. IMDB) are already well analyzed and should be avoided, unless you want to try to take a very different angle or use innovative analysis methods

## Develop a narrative

In the early stages of the analysis process, try to uncover patterns to help you form and shape a narrative through-line for the explainer

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## FINAL PROJECT GUIDELINES

### Design visualization interactions

Choose base visualizations that can support a high level of interactivity  
Bubble charts, tree maps, and word clouds typically aren't the most effective choices

Design interactive features that would enable viewers to interact with the data in a way that strengthens your narrative

Tooltip is typically not enough interaction

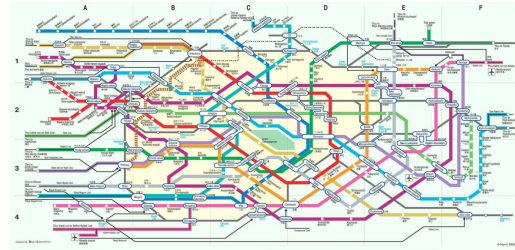
Draw inspiration from sites like the New York Times and the Pudding

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## NETWORK ANALYSIS

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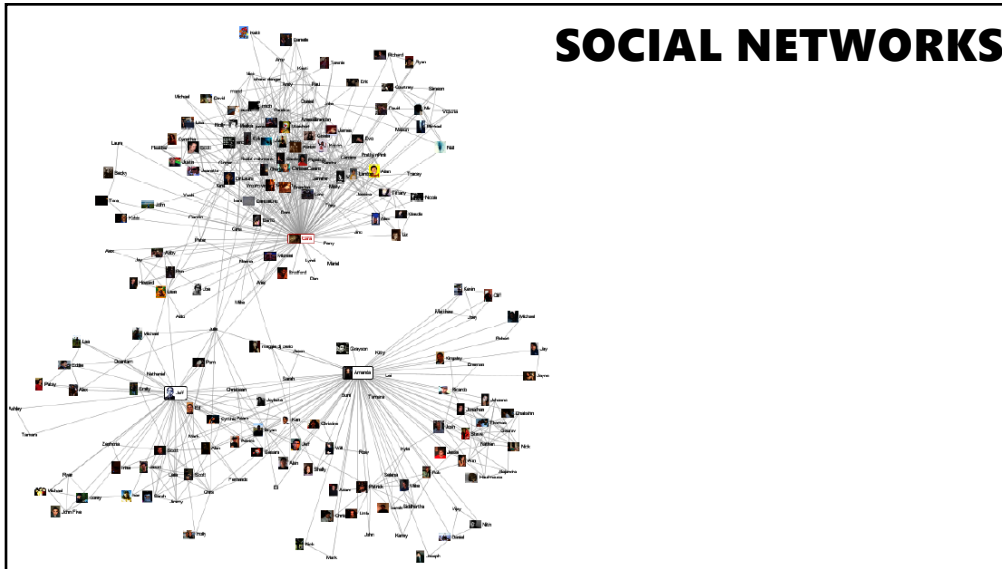
# TRANSPORTATION



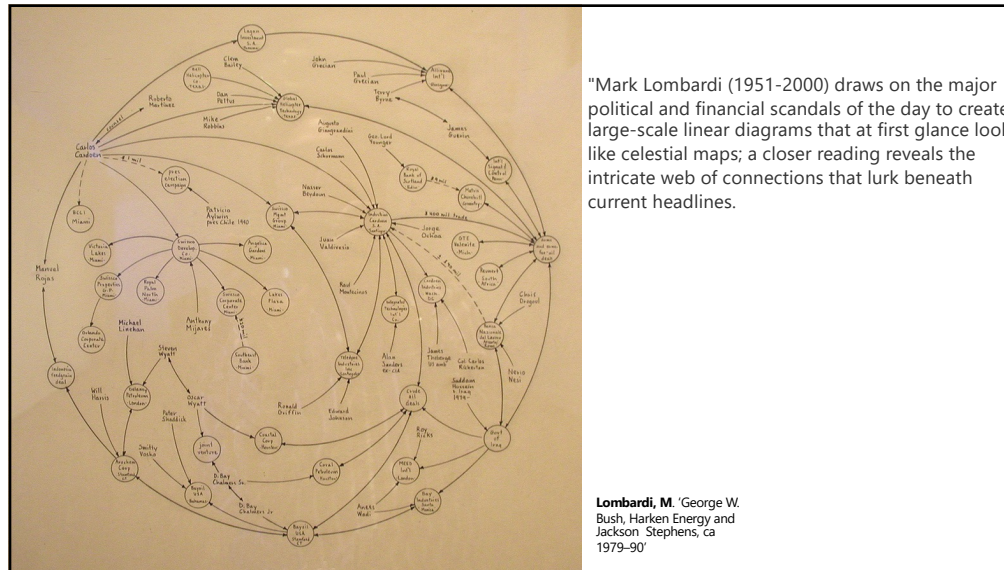
<http://www.ix97.com/maps/>

70

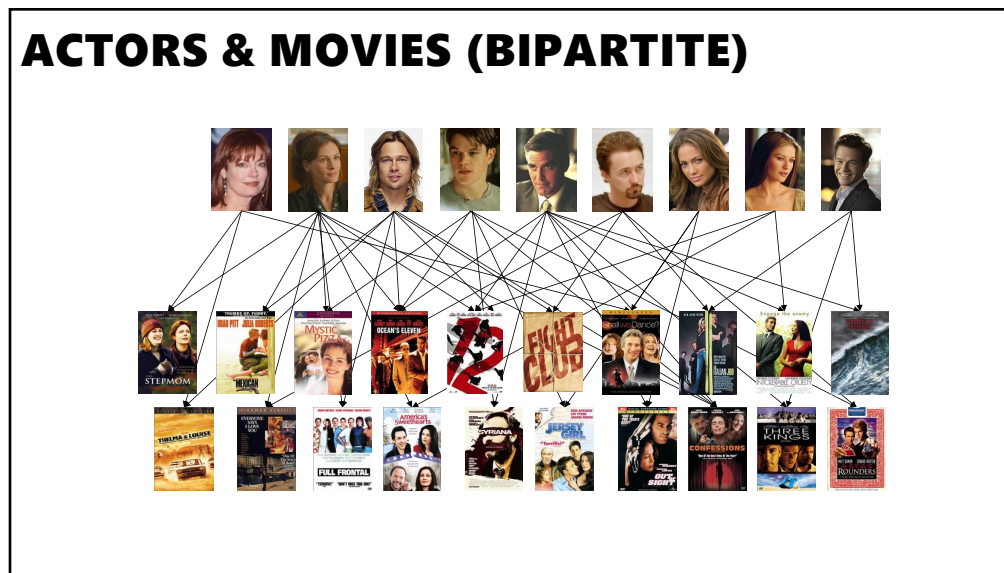
# SOCIAL NETWORKS



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visual complexity

Search the VC database:  GO

Latest Projects: 

Indexing 1000 projects

Filter by: SUBJECT

- Art (74)
- Biology (60)
- Business Networks (30)
- Computer Systems (39)
- Food Webs (16)
- Internet (35)
- Knowledge Networks (141)
- Multi-Domain Representation (70)
- Music (47)
- Others (77)
- Pattern Recognition (53)
- Political Networks (34)
- Semantic Networks (44)
- Social Networks (135)
- Transportation Networks (70)
- World Wide Web (55)

See All (1000)

visual complexity Mapping Patterns of Information Buy now

visual complexity aStore

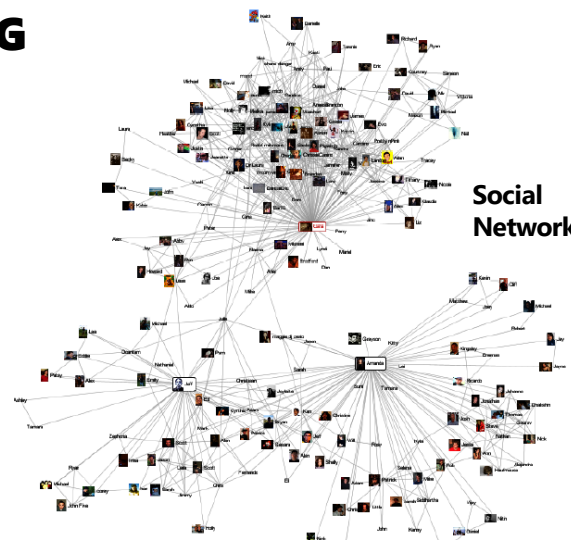
See all recommended books

<http://www.visualcomplexity.com/vc/>

75

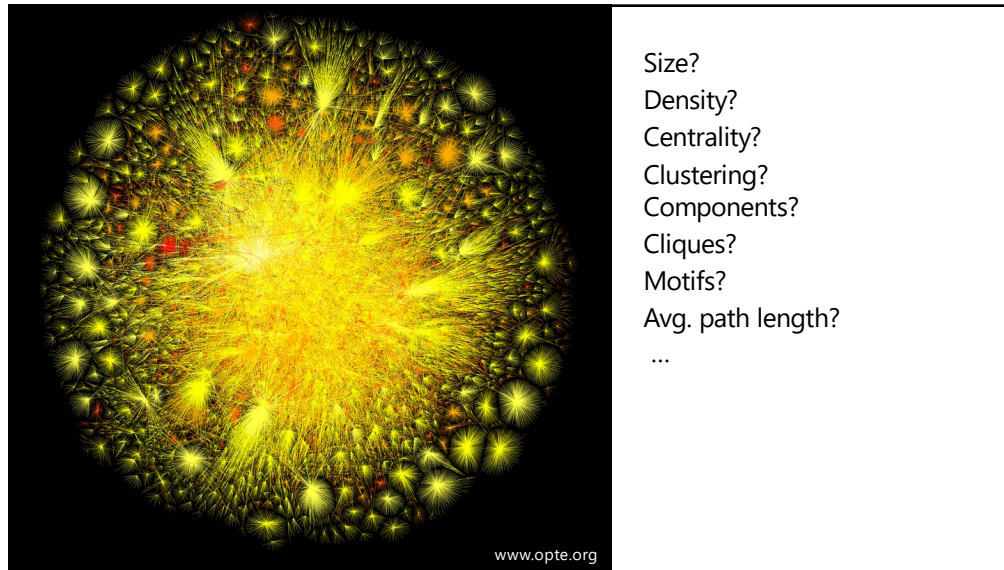
# CHARACTERIZING NETWORKS

What does it look like?



**Social Network**

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**TODAY**

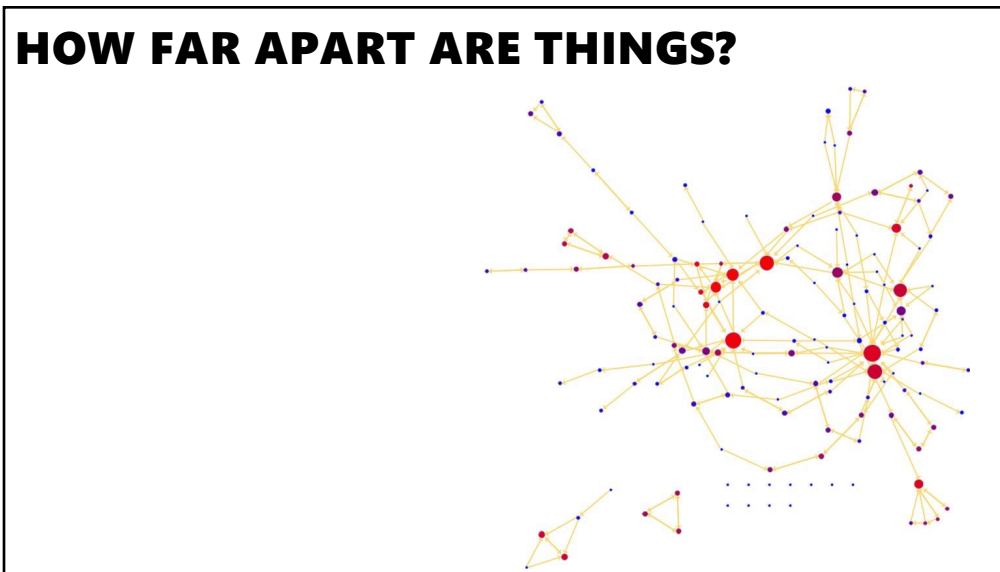
Learning Objectives

1. Measures of importance/centrality
2. Extracting community structure
3. Simulating network models

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# CENTRALITY

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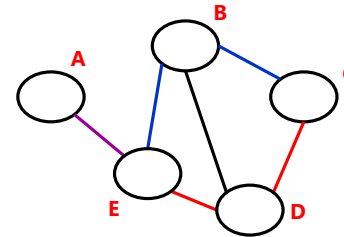
## DISTANCE: SHORTEST PATHS

### Shortest path (geodesic path)

The shortest sequence of links connecting two nodes  
Not always unique

A and C are connected by 2 shortest paths

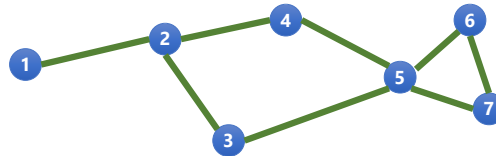
A - E - B - C  
A - E - D - C



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## DISTANCE: SHORTEST PATHS

Shortest path from 2 to 3: 1

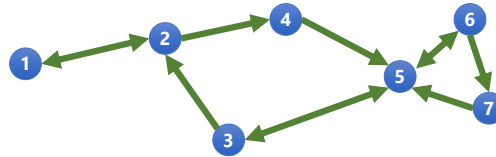


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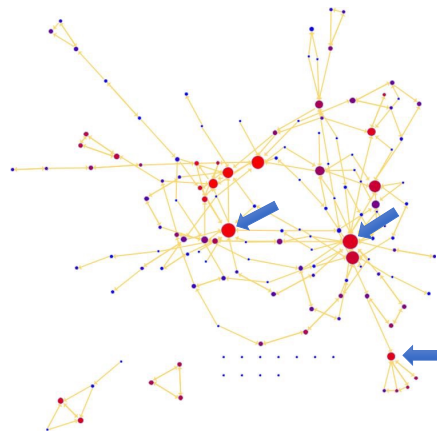
## DISTANCE: SHORTEST PATHS

Shortest path from 2 to 3?



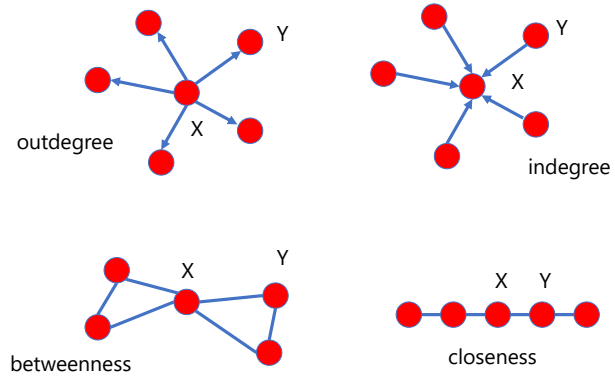
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## MOST IMPORTANT NODE



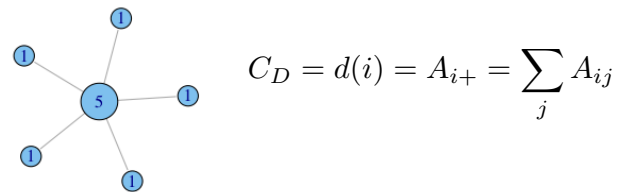
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## CENTRALITY



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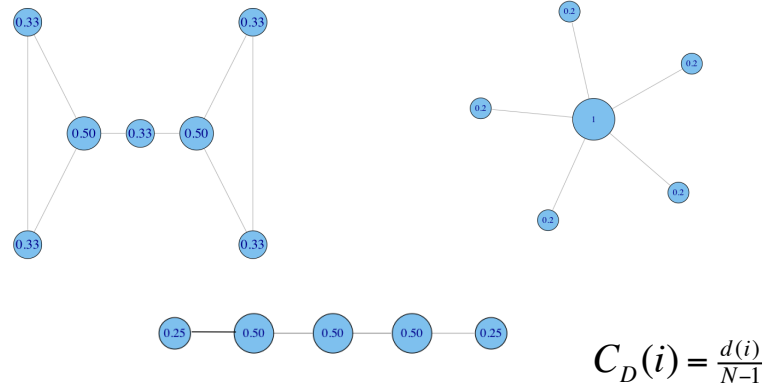
## DEGREE CENTRALITY (UNDIRECTED)



$$C_D = d(i) = A_{i+} = \sum_j A_{ij}$$

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## NORMALIZED DEGREE CENTRALITY



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## WHEN IS DEGREE NOT SUFFICIENT?

### Does not capture

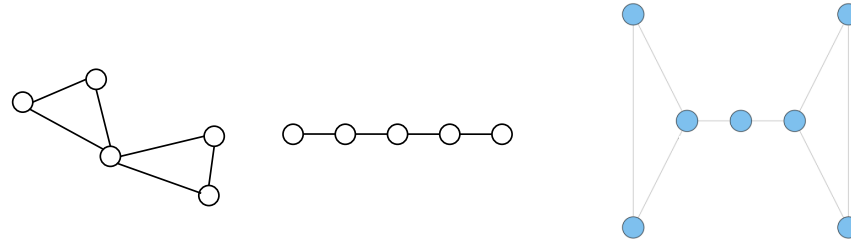
Ability to broker between groups

Likelihood that information originating anywhere in the network reaches you

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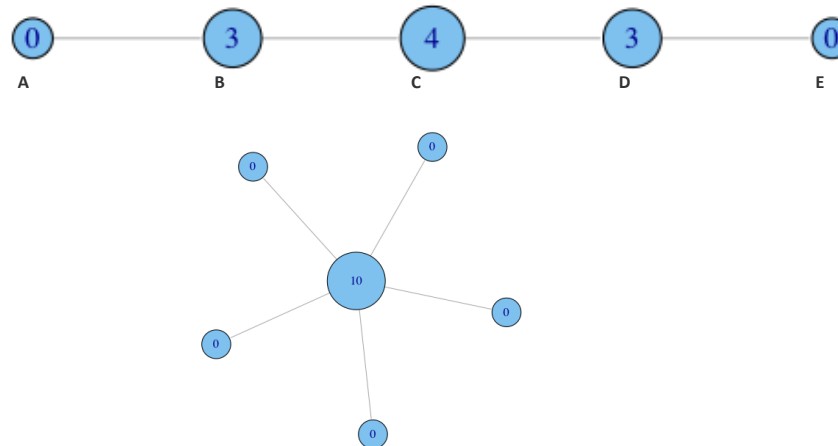
## BETWEENNESS

Assuming nodes communicate using the most direct (shortest) route, how many pairs of nodes have to pass information through target node?



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## BETWEENNESS - EXAMPLES



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## BETWEENNESS - DEFINITION

$$C_B(i) = \sum_{j,k \neq i, j < k} g_{jk}(i) / g_{jk}$$

$g_{jk}$  = the number of shortest paths connecting  $jk$   
 $g_{jk}(i)$  = the number of shortest paths containing  $i$ .

Normalization:

$$C'_B(i) = C_B(i) / [(n-1)(n-2)/2]$$

 number of pairs of vertices  
excluding the vertex itself

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## WHEN ARE $C_d$ , AND $C_b$ NOT SUFFICIENT?

### Does not capture

Likelihood that information originating anywhere in the network reaches you

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## CLOSENESS - DEFINITION

e.g., which node is closest to the *center* of the graph

Closeness Centrality:

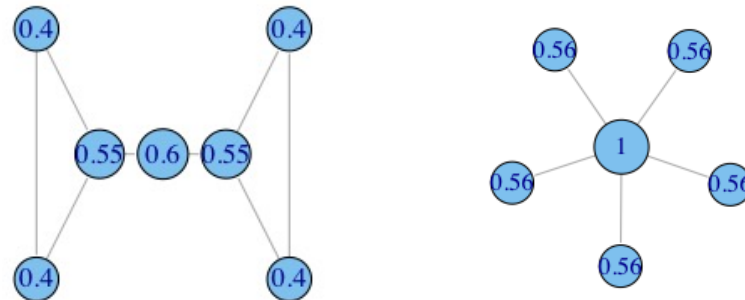
$$C_c(i) = \left[ \sum_{j=1, j \neq i}^N d(i, j) \right]^{-1}$$

Normalized Closeness Centrality

$$C'_c(i) = (C_c(i)) / (N - 1) = \frac{N - 1}{\sum_{j=1, j \neq i}^N d(i, j)}$$

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## CLOSENESS - EXAMPLES

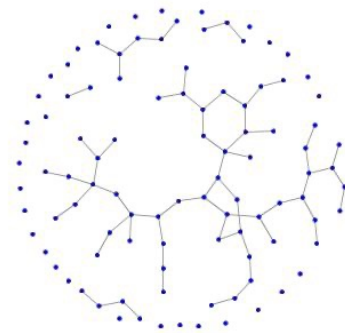


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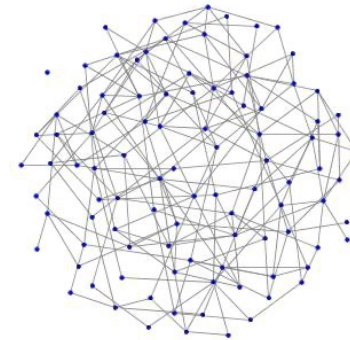
# COMMUNITY STRUCTURE

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## HOW DENSE IS IT?



density =  $e / e_{\max}$



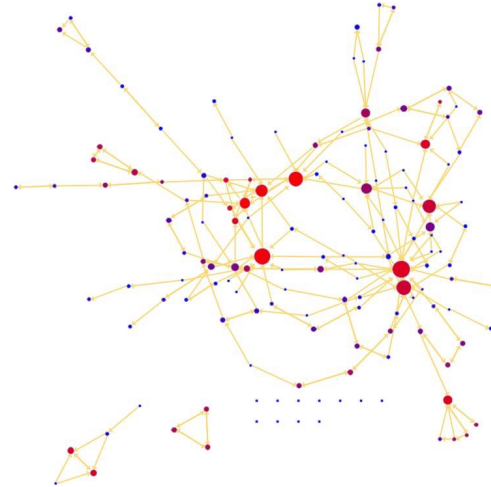
Max. possible edges:

Directed:  $e_{\max} = n*(n-1)$

Undirected:  $e_{\max} = n*(n-1)/2$

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## IS EVERYTHING CONNECTED?



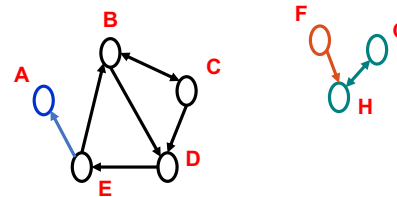
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## CONNECTED COMPONENTS - DIRECTED

### Strongly connected components

Each node in component can be reached from every other node in component by following directed links

**B C D E**  
**A**  
**G H**  
**F**



### Weakly connected components

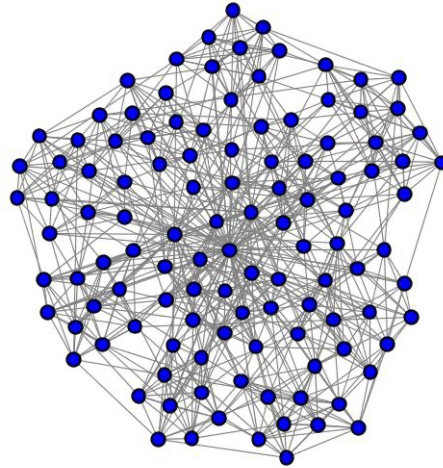
Each node can be reached from every other node by following links in either direction

**A B C D E**  
**G H F**

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## COMMUNITY FINDING - CLUSTERING



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## HIERARCHICAL CLUSTERING

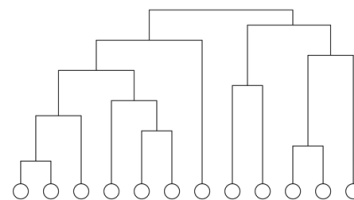
### Process

Calculate affinity weights  $W$  for all pairs of vertices

**Start:**  $N$  disconnected vertices

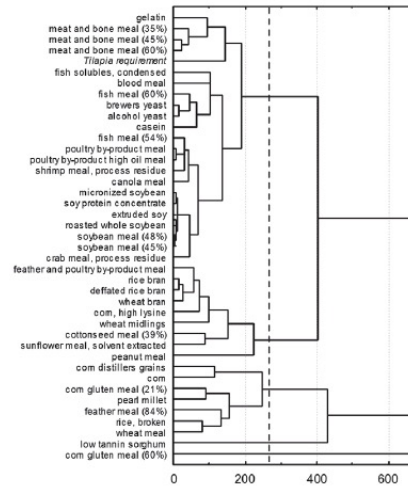
Add edges (one by one) between pairs of clusters in order of decreasing weight (use closest distance to compare clusters)

**Result:** nested components



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## CLUSTER DENDROGRAMS

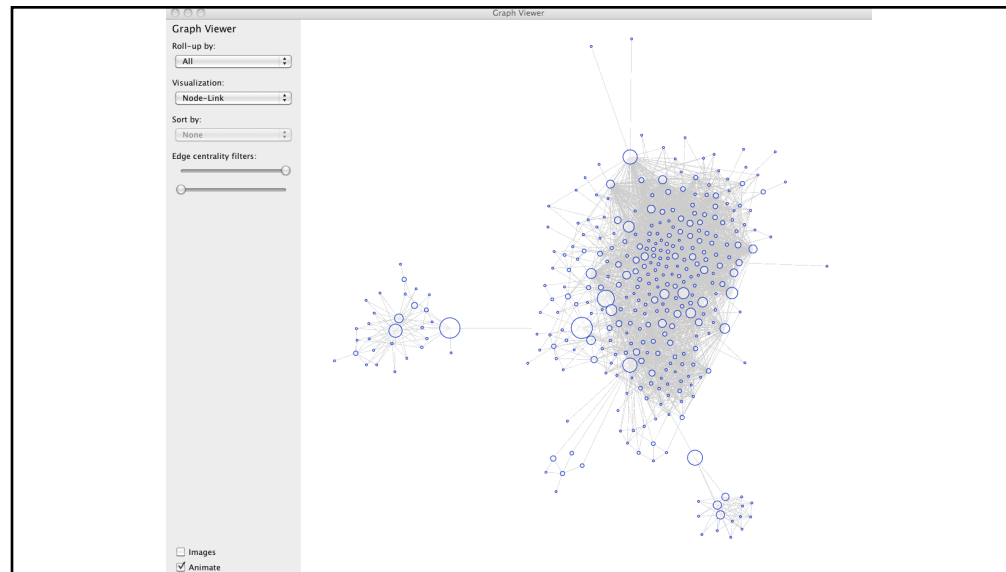


Depicts cluster trees produced by hierarchical clustering algorithms

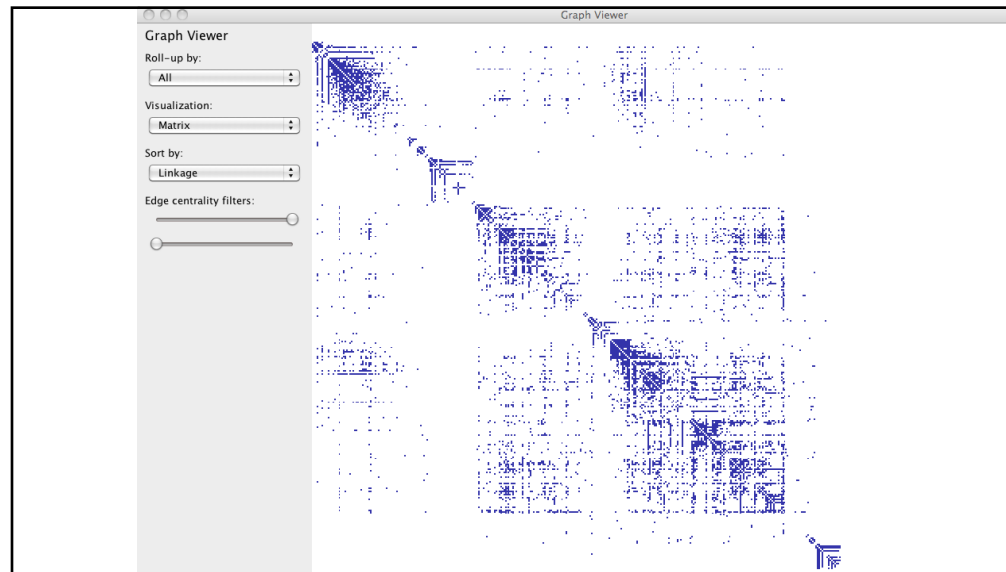
Leaf nodes arranged in line, internal node depth indicates order/value at which clusters merge

Basic recursive layout with orthogonal two-segment edges

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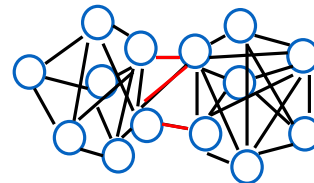
## BETWEENNESS CLUSTERING

### Girvan and Newman 2002 iterative algorithm:

Compute  $C_b$  of all edges

Remove edge  $i$  where  $C_b(i) == \max(C_b)$

Recalculate betweenness



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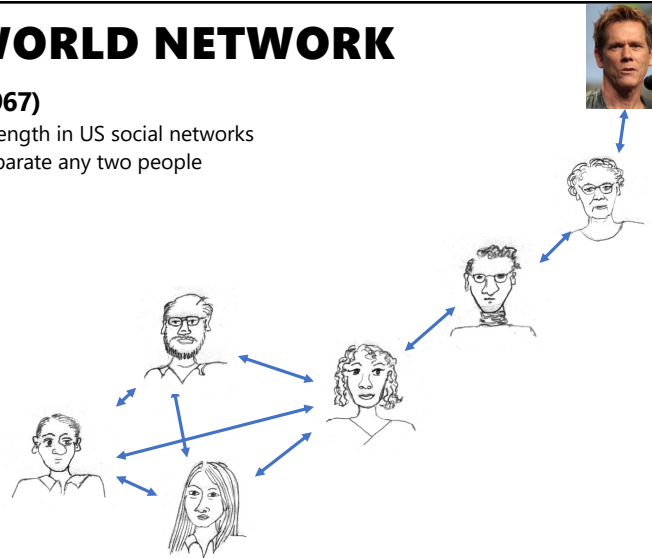
# SIMULATING NETWORK MODELS

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## SMALL WORLD NETWORK

**Milgram (1967)**

Mean path length in US social networks  
~ 6 hops separate any two people

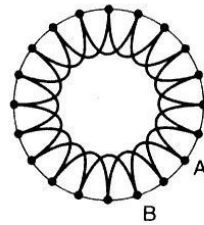


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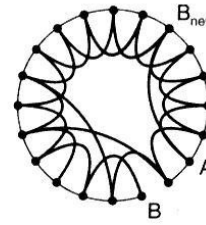
## SMALL WORLD NETWORK

**Watts and Strogatz 1998**

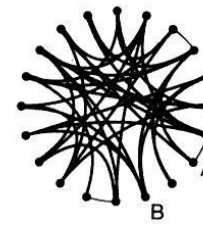
a few random links in otherwise structured graph make network a small world



regular lattice:  
my friend's friend is  
always my friend



small world:  
mostly structured  
with a few random  
connections



random graph:  
all connections  
random

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## DEFINING SMALL WORLD PHENOMENA

### Properties

high clustering  
low mean shortest path

$$C_{\text{network}} \gg C_{\text{random graph}}$$

$$l_{\text{network}} \approx \ln(N)$$

### Examples

neural network of *C. elegans*  
semantic networks of languages  
actor collaboration graph  
food webs

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## **SUMMARY**

### **Structural analysis**

- Centrality
- Community structure

**Simulation models enable further analysis**

**Network analysis applicable in many domains**

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