Network Analysis

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CS 448B: Visualization
Fall 2018

Announcements

Final project

New visualization research or data analysis

- Pose problem, Implement creative solution
- Design studies/evaluations

Deliverables

- Implementation of solution
- 6-8 page paper in format of conference paper submission
- Project progress presentations

Schedule

- Project proposal: Mon 11/5
- Project progress presentation: 11/12 and 11/14 in class (3-4 min)
- Final poster presentation: 12/5 Location: Lathrop 282
- Final paper: 12/9 11:59pm

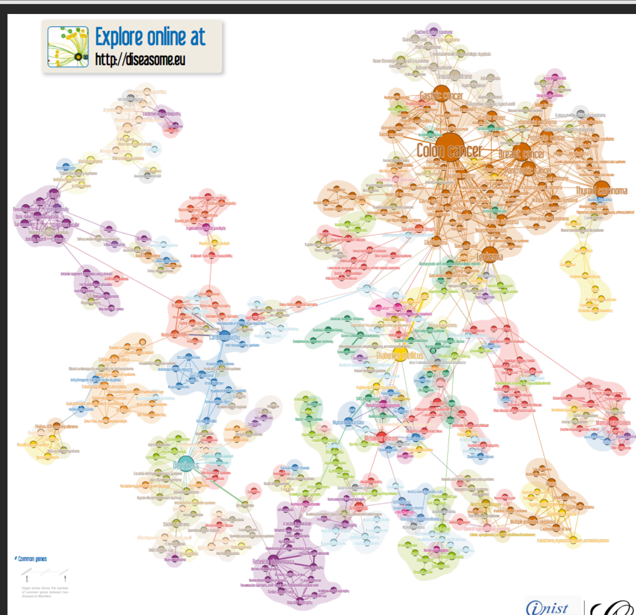
Grading

- Groups of up to 3 people, graded individually
- Clearly report responsibilities of each member

Network Analysis

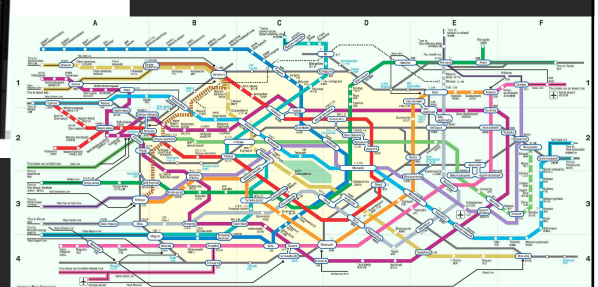
*Slides adapted from E. Adar's / L. Adamic's Network Theory and Applications course slides.

Diseases

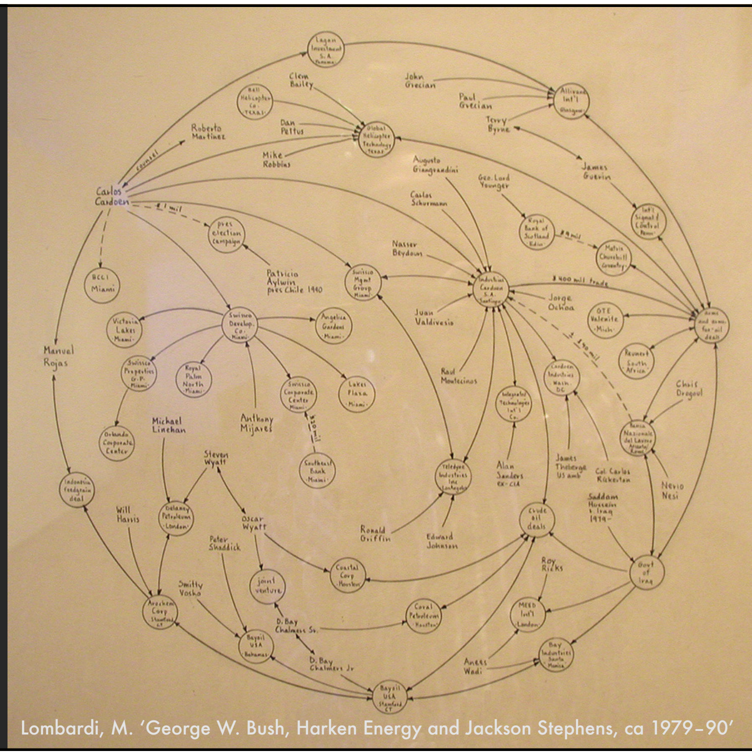


http://diseasome.eu/

Transportation

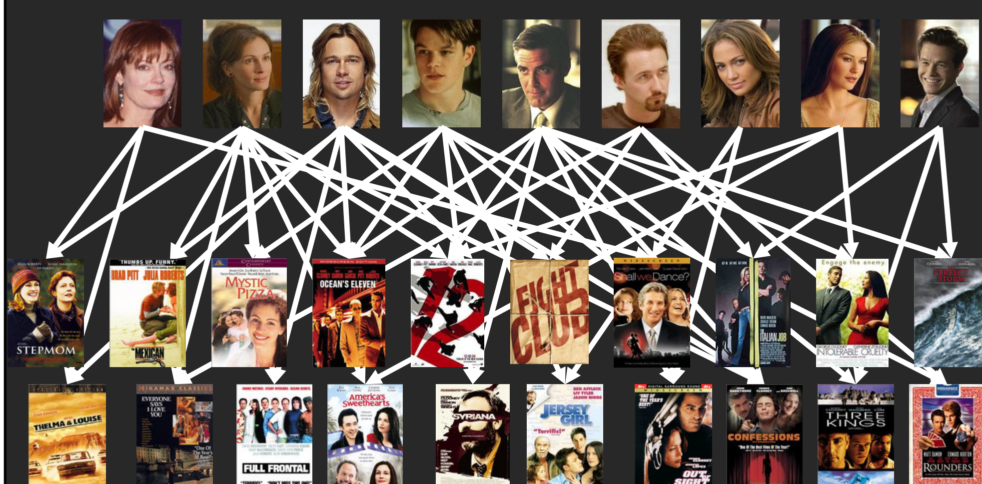


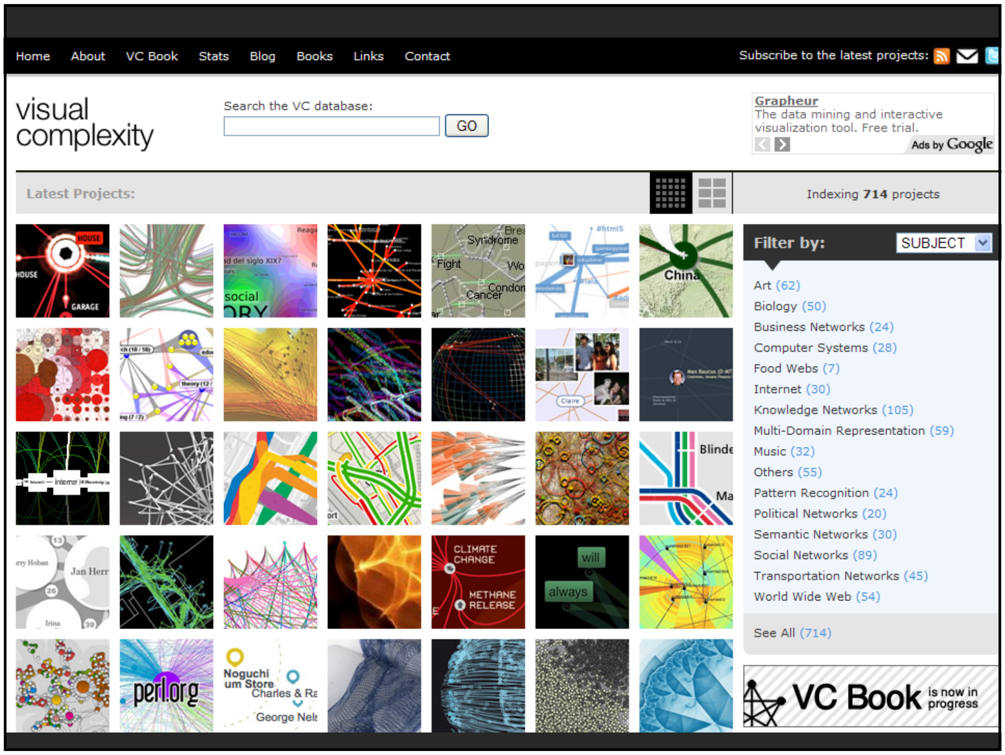
http://www.lx97.com/maps/



Lombardi, M. 'George W. Bush, Harken Energy and Jackson Stephens, ca 1979–90'

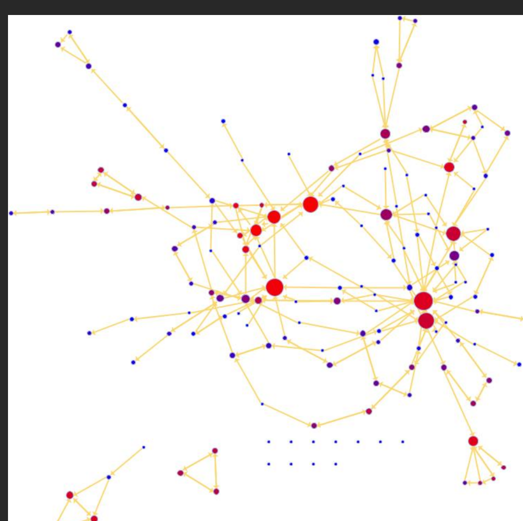
Actors and movies (bipartite)



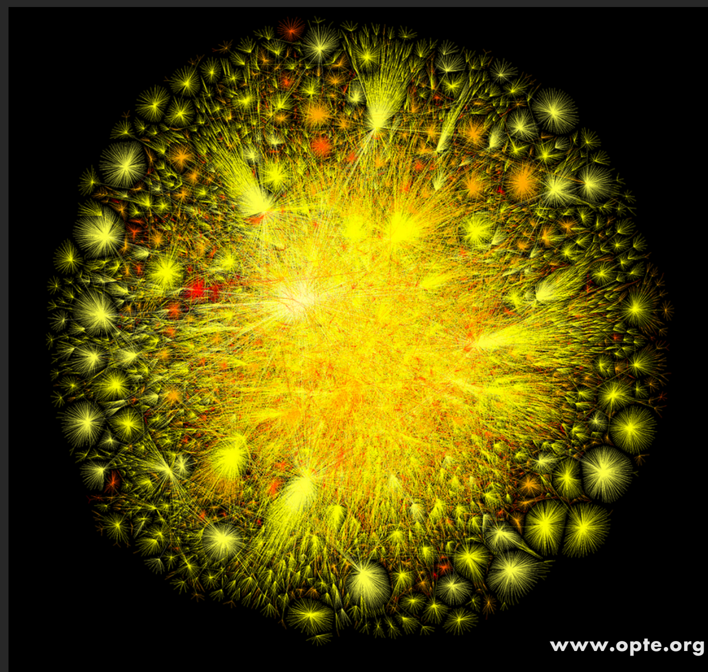


Characterizing networks

What does it look like?



Size?
Density?
Centralization?
Clustering?
Components?
Cliques?
Motifs?
Avg. path length?
...



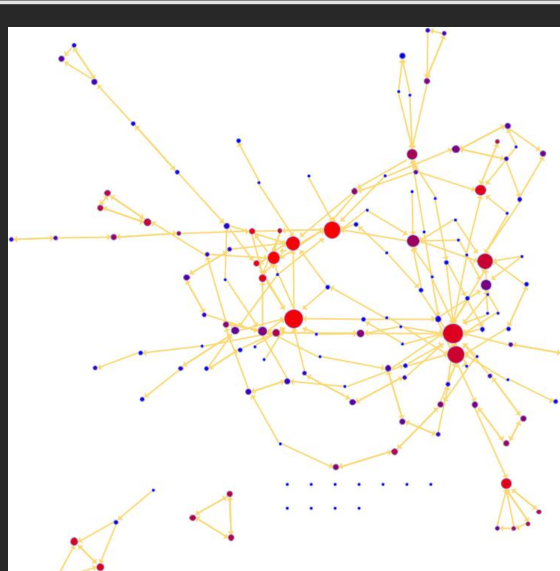
Topics

Network Analysis

- Centrality / centralization
- Community structure
- Pattern identification
- Models

Centrality

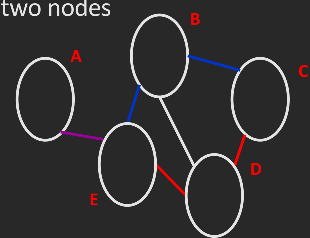
How far apart are things?



Distance: shortest paths

Shortest path (geodesic path)

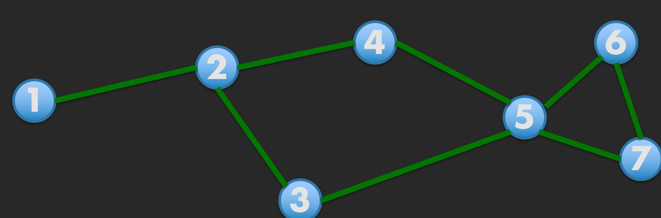
- The shortest sequence of links connecting two nodes
- Not always unique



- A and C are connected by 2 shortest paths
  - A – E – B - C
  - A – E – D - C

Distance: shortest paths

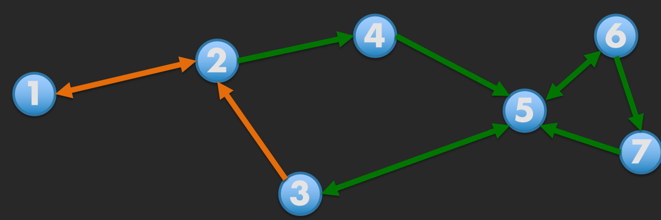
Shortest path from 2 to 3: 1



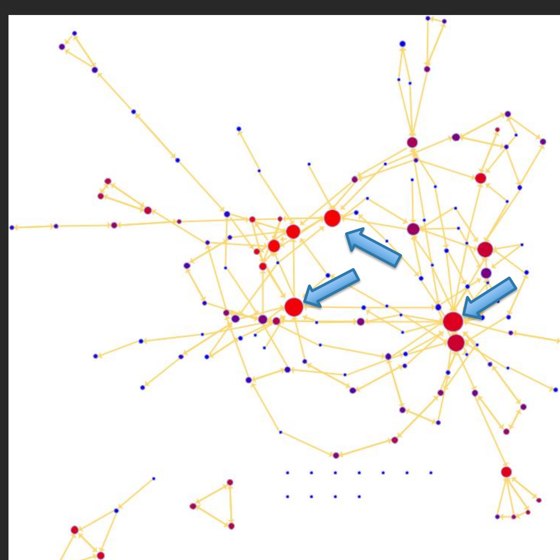
Diameter?

Distance: shortest paths

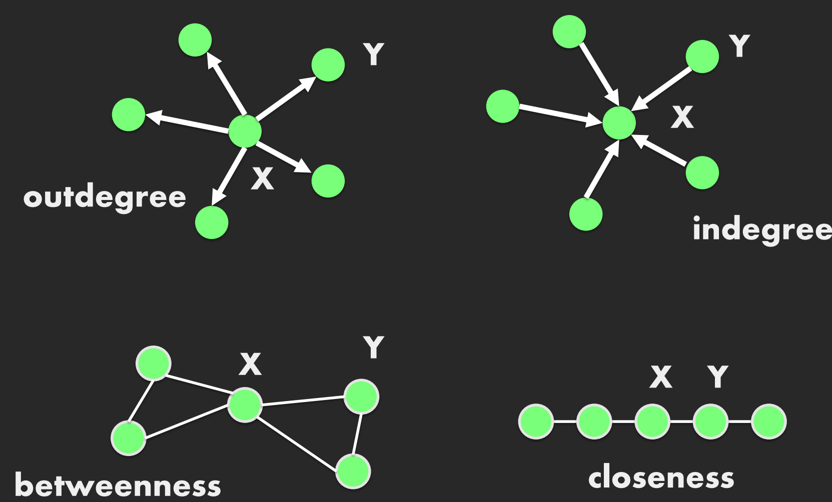
Shortest path from 2 to 3?



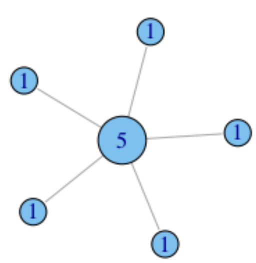
Most important node?



Centrality

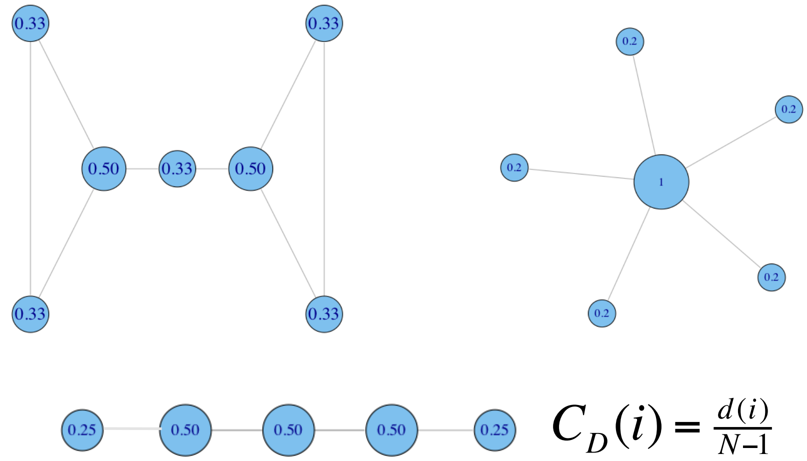


Degree centrality (undirected)



$$C_D = d(n_i) = A_{i+} = \sum_j A_{ij}$$

Normalized degree centrality



$$C_D(i) = \frac{d(i)}{N-1}$$

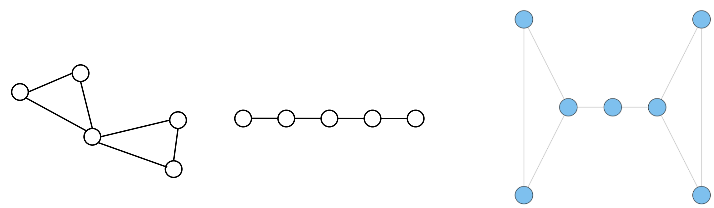
When is degree not sufficient?

Does not capture

- Ability to broker between groups
- Likelihood that information originating anywhere in the network reaches you

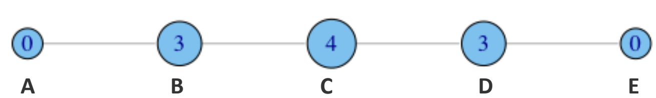
Betweenness

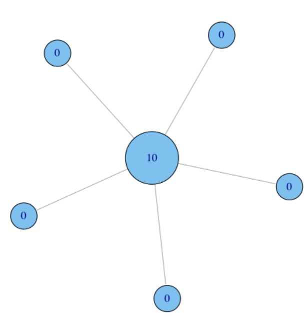
Assuming nodes communicate using the most direct (shortest) route, how many pairs of nodes have to pass information through target node?



Betweenness - examples

non-normalized:





Betweenness: definition

$$C_B(i) = \sum_{j,k \neq i, j<k} g_{jk}(i) / g_{jk}$$

g_{jk} = the number of geodesics connecting *jk*
g_{jk}(i) = the number that node *i* is on.

Normalization:

$$C'_B(i) = C_B(i) / [(n-1)(n-2)/2]$$

number of pairs of vertices excluding the vertex itself

When are C_d, C_b not sufficient?

Do not capture

Likelihood that information originating anywhere in the network reaches you

Closeness: definition

Being close to the center of the graph

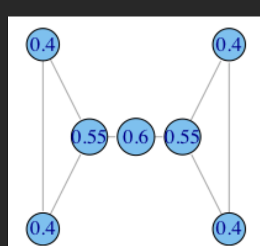
Closeness Centrality:

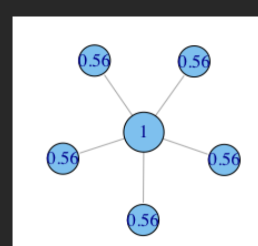
$$C_c(i) = \left[\sum_{j=1, j \neq i}^{N} d(i,j) \right]^{-1}$$

Normalized Closeness Centrality

$$C_C^{'}(i) = (C_C(i)) / (N-1) = \frac{N-1}{\sum_{j=1, j \neq i}^{N} d(i,j)}$$

Examples - closeness





Centrality in directed networks

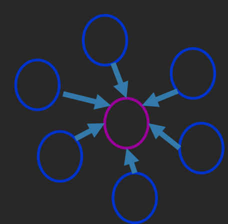
Prestige ~ indegree centrality

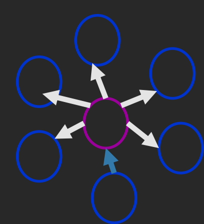
Betweenness ~ consider directed shortest paths

Closeness ~ consider nodes from which target node can be reached

Influence range ~ nodes reachable from target node

Straight-forward modifications to equations for non-directed graphs





Characterizing nodes

- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

	Low Degree	Low Closeness	Low Betweenness
High Degree		Node embedded in cluster that is far from the rest of the network	Node's connections are redundant - communication bypasses him/her
High Closeness	Node links to a small number of important/active other nodes.		Many paths likely to be in network; node is near many people, but so are many others
High Betweenness	Node's few ties are crucial for network flow	Rare. Node monopolizes the ties from a small number of people to many others.	

Centralization – how equal

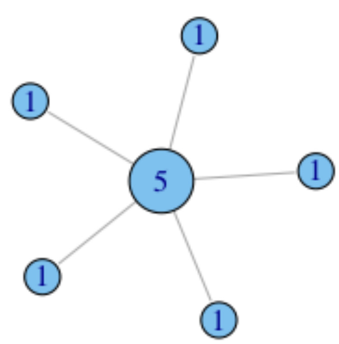
Variation in the centrality scores among the nodes

Freeman's general formula for centralization:

maximum value in the network

$$C_D = \frac{\sum_{i=1}^{g}\left[C_D(n^*) - C_D(i)\right]}{[(N-1)(N-2)]}$$

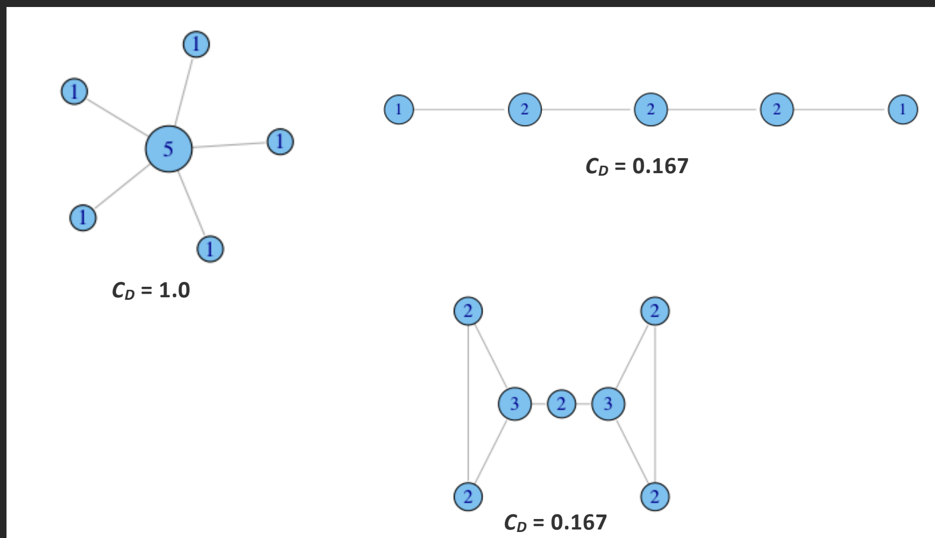
Examples



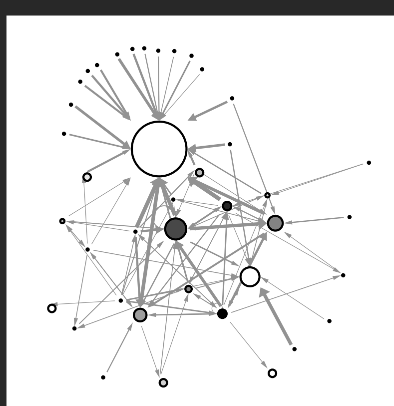
$$C_D = \frac{\sum_{i=1}^{g}\left[C_D(n^*) - C_D(n_i)\right]}{[(N-1)(N-2)]}$$

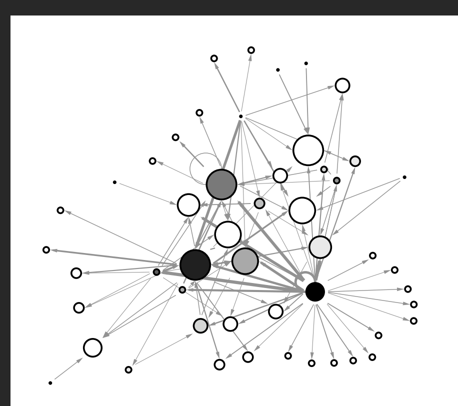
$$C_D = \frac{(5-5)+(5-1)\times 5}{(6-1)(6-2)} = 1$$

Examples



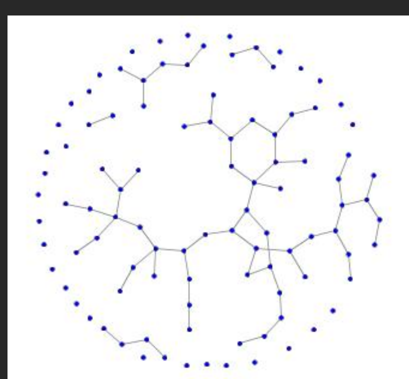
Financial networks

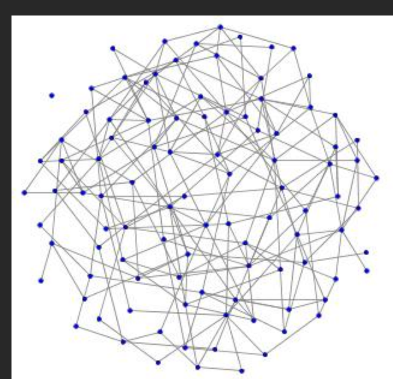




Community Structure

How dense is it?

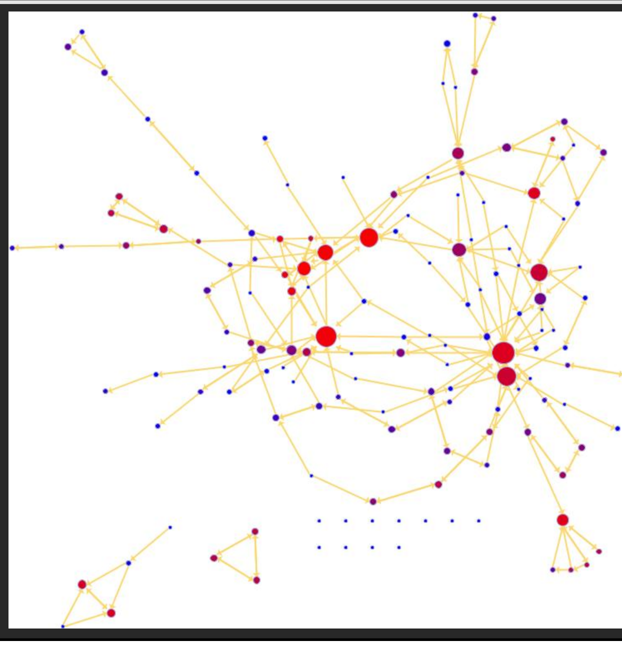




density = e / e_{max}

Max. possible edges:
- Directed: $e_{max} = n*(n-1)$
- Undirected: $e_{max} = n*(n-1)/2$

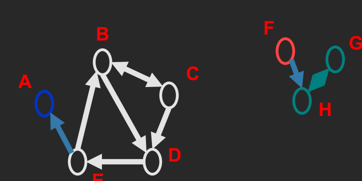
Is everything connected?



Connected Components - Directed

Strongly connected components

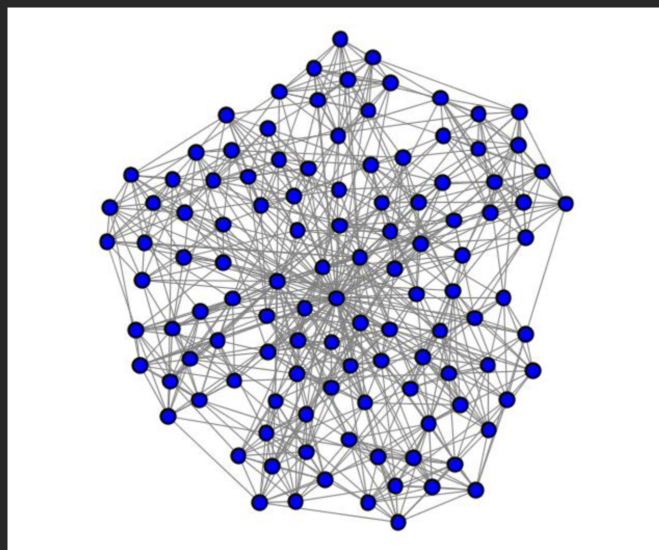
- Each node in component can be reached from every other node in component by following directed links
  - B C D E
  - A
  - G H
  - F



Weakly connected components

- Each node can be reached from every other node by following links in either direction
  - A B C D E
  - G H F

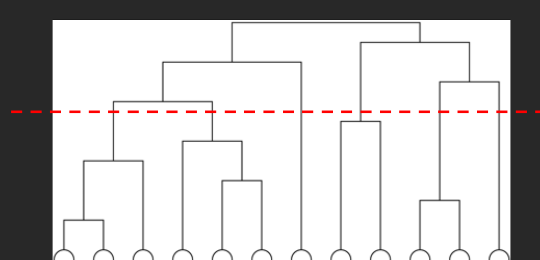
Community finding (clustering)



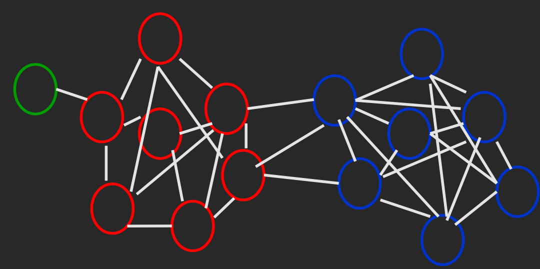
Hierarchical clustering

Process:

- Calculate affinity weights W for all pairs of vertices
- Start: N disconnected vertices
- Adding edges (one by one) between pairs of clusters in order of decreasing weight (use closest distance to compare clusters)
- Result: nested components



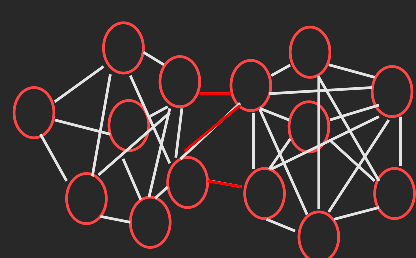
Hierarchical clustering (path counts)



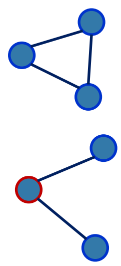
Betweenness clustering

Girvan and Newman 2002 iterative algorithm:

- Compute C_b of all *edges*
- Remove edge *i* where $C_b(i) == max(C_b)$
- Recalculate betweenness

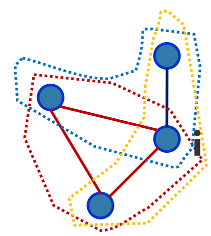


Clustering coefficient



Local clustering coefficient:

$$C_i = \frac{\text{number of closed triplets centered on i}}{\text{number of connected triplets centered on i}}$$



Global clustering coefficient:

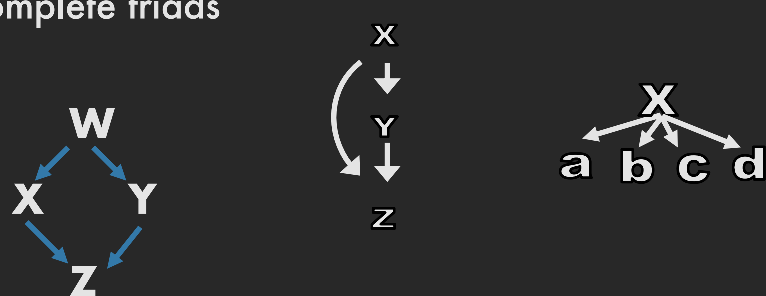
$$C_G = \frac{3\text{* number of closed triplets}}{\text{number of connected triplets}}$$

$C_i = 1/3 = 0.33$

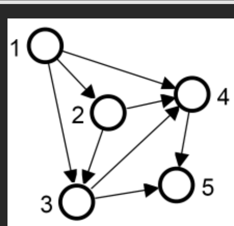
$C_G = 3*1/5 = 0.6$

Pattern finding - motifs

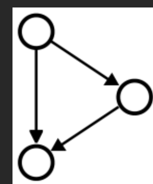
Define / search for a particular structure, e.g. complete triads



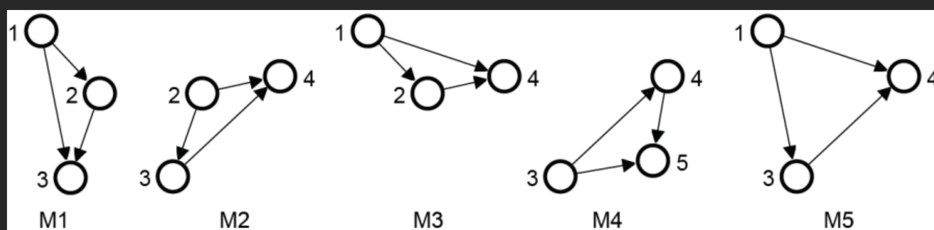
Motifs can overlap in the network



graph



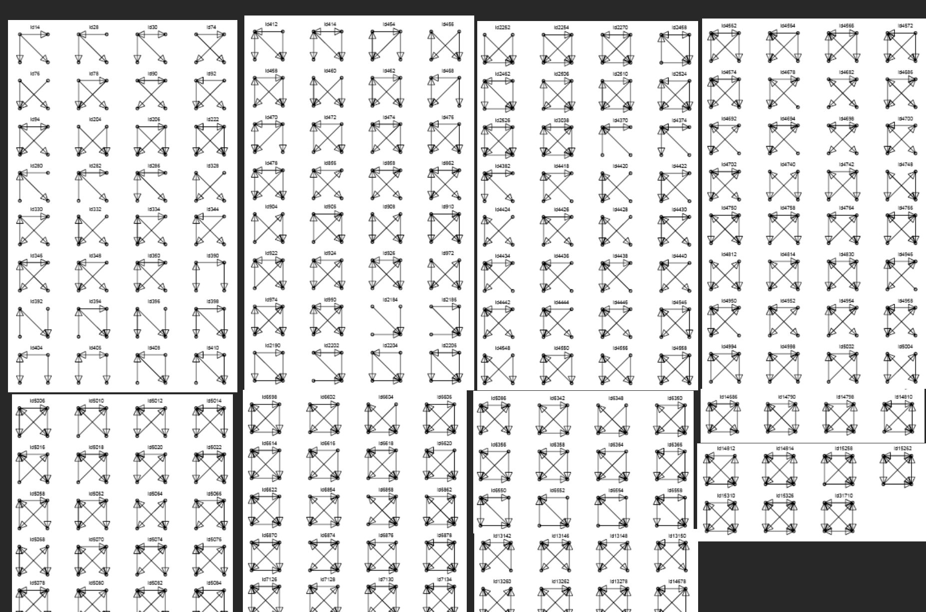
motif to be found



motif matches

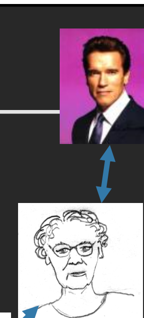
http://mavisto.ipk-gatersleben.de/frequency_concepts.html

4 node subgraphs



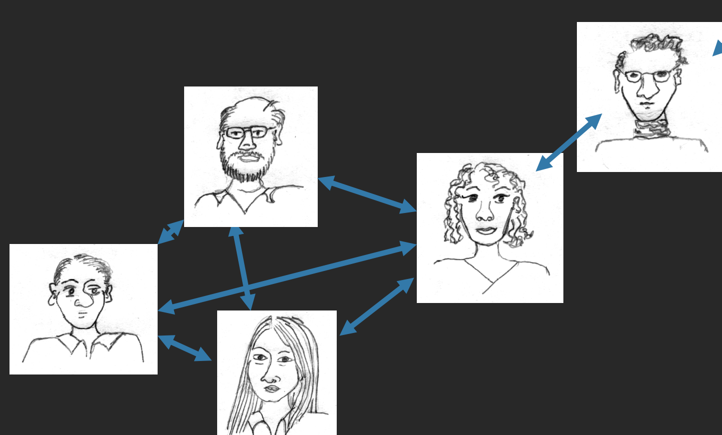
Simulating network models

Small world network



Milgram (1967)

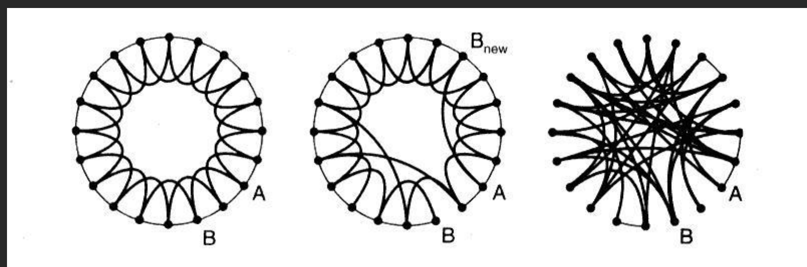
- Mean path length in US social networks
- ~ 6 hops separate any two people



Small world networks

Watts and Strogatz 1998

- a few random links in an otherwise structured graph make the network a small world



regular lattice: my friend's friend is always my friend

small world: mostly structured with a few random connections

random graph: all connections random

Defining small world phenomenon

Pattern:

- high clustering
- low mean shortest path

$$C_{\text{network}} >> C_{\text{random graph}}$$

$$l_{\text{network}} \approx \ln(N)$$

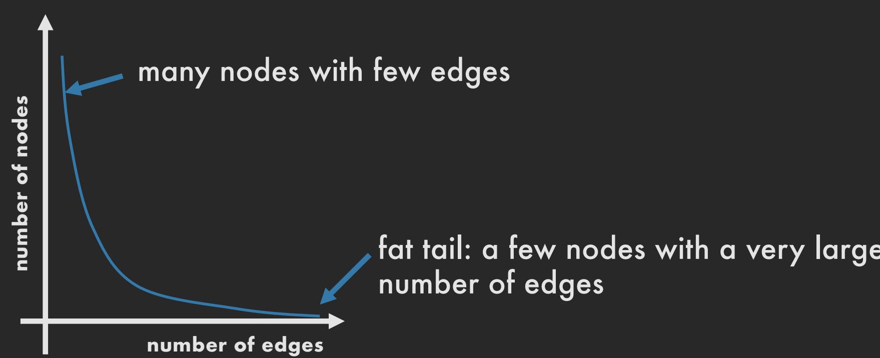
Examples

- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph
- food webs

Power law networks

Many real world networks contain hubs: highly connected nodes

Usually the distribution of edges is extremely skewed



Summary

Structural analysis

- Centrality
- Community structure
- Pattern finding

→ Widely applicable across domains