Graph Layout

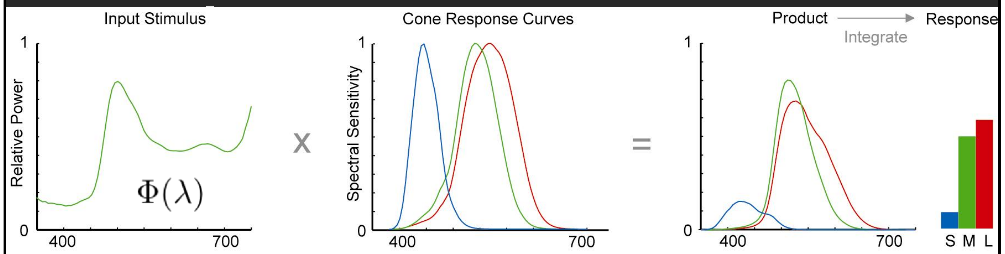
Maneesh Agrawala

CS 448B: Visualization
Fall 2018

Last Time: Color

Computing Cone Response

Integrate cone response with input



$$L = \int \Phi(\lambda) L(\lambda) \mathrm{d}\lambda$$

$$M = \int \Phi(\lambda) M(\lambda) \mathrm{d}\lambda$$

$$S = \int \Phi(\lambda) S(\lambda) \mathrm{d}\lambda$$

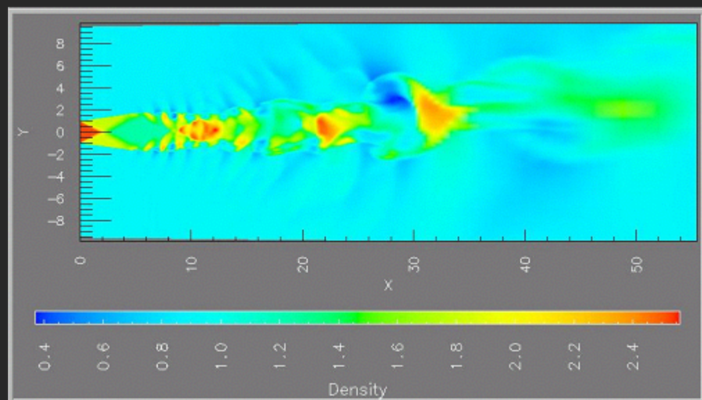
Palette Design + Color Names

Minimize overlap and ambiguity of color names

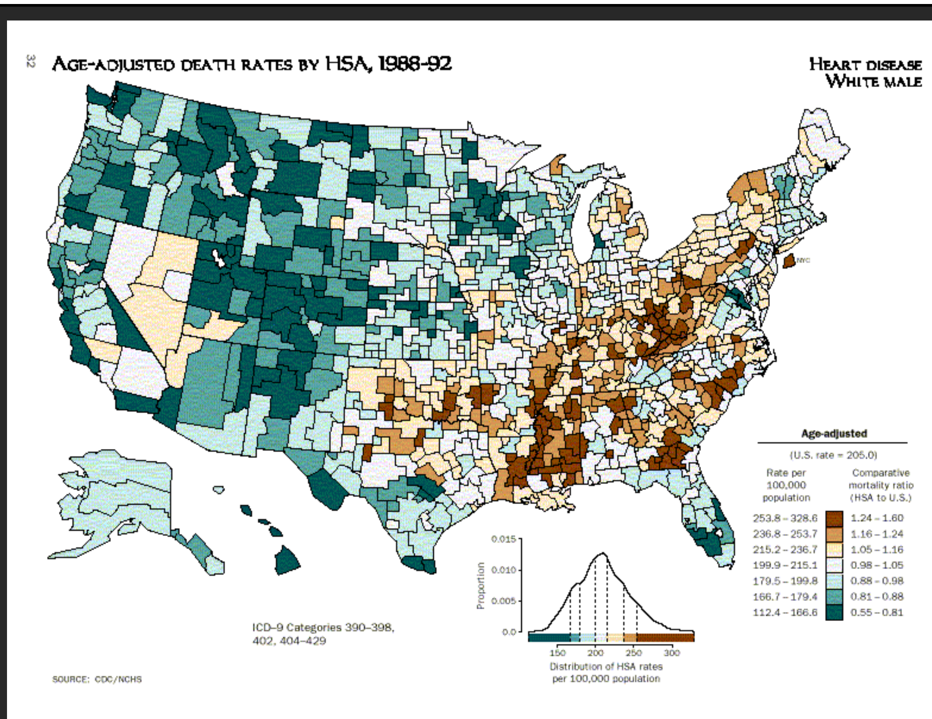
Color Name Distance										Salience	Name
0.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	0.20	.47	**blue** 62.9%
1.00	0.00	1.00	0.97	1.00	1.00	1.00	1.00	0.96	1.00	.90	**orange** 93.9%
1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	0.90	0.99	.67	**green** 79.8%
1.00	0.97	1.00	0.00	1.00	0.95	0.99	1.00	1.00	1.00	.66	**red** 80.4%
0.98	1.00	1.00	1.00	0.00	0.96	0.91	0.97	1.00	0.99	.47	**purple** 51.4%
1.00	1.00	1.00	0.95	0.96	0.00	0.97	0.93	0.98	1.00	.37	**brown** 54.0%
1.00	1.00	1.00	0.99	0.91	0.97	0.00	1.00	1.00	1.00	.58	**pink** 71.7%
1.00	1.00	1.00	1.00	0.97	0.93	1.00	0.00	1.00	1.00	.67	**grey** 79.4%
1.00	0.96	0.90	1.00	1.00	0.98	1.00	1.00	0.00	1.00	.18	**yellow** 31.2%
0.20	1.00	0.99	1.00	0.99	1.00	1.00	1.00	1.00	0.00	.25	**blue** 25.4%
Tableau-10								*Average*	*0.97*	*.52*	

http://vis.stanford.edu/color-names

Avoid rainbow color maps!



1. People segment colors into classes
2. Hues are not naturally ordered
3. Different lightness emphasizes certain scalar values
4. Low luminance colors (blue) hide high frequencies



Announcements

Final project

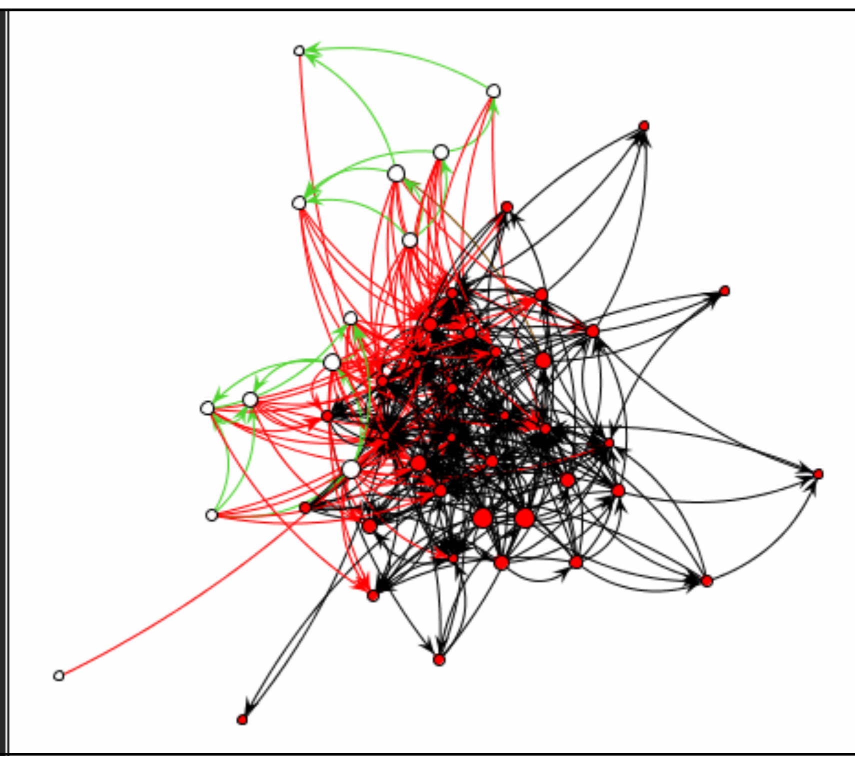
New visualization research or data analysis
- Pose problem, Implement creative solution
- Design studies/evaluations

Deliverables
- Implementation of solution
- 6-8 page paper in format of conference paper submission
- Project progress presentations

Schedule
- Project proposal: Mon 11/5
- Project progress presentation: 11/12 and 11/14 in class (3-4 min)
- Final poster presentation: 12/5 Location: Lathrop 282
- Final paper: 12/9 11:59pm

Grading
- Groups of up to 3 people, graded individually
- Clearly report responsibilities of each member

Graph Layout

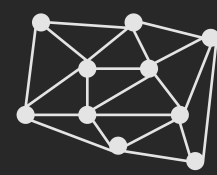


Graphs and Trees

Graphs

Model relations among data

Nodes and *edges*

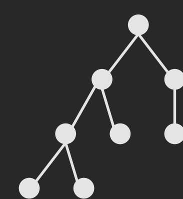


Trees

Graphs with hierarchical structure

- Connected graph with N-1 edges

Nodes as *parents* and *children*



Spatial Layout

Primary concern – layout of nodes and edges

Often (but not always) goal is to depict structure

- Connectivity, path-following
- Network distance
- Clustering
- Ordering (e.g., hierarchy level)

Applications

Organization Charts
Genealogy
Diagramming (e.g., Visio)
Biological Interactions (Genes, Proteins)
Computer Networks
Social Networks
Tournaments
Simulation and Modeling
Integrated Circuit Design

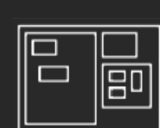
Tree Visualization

Indentation

- Linear list, indentation encodes depth

Node-Link diagrams

- Nodes connected by lines/curves



Enclosure diagrams

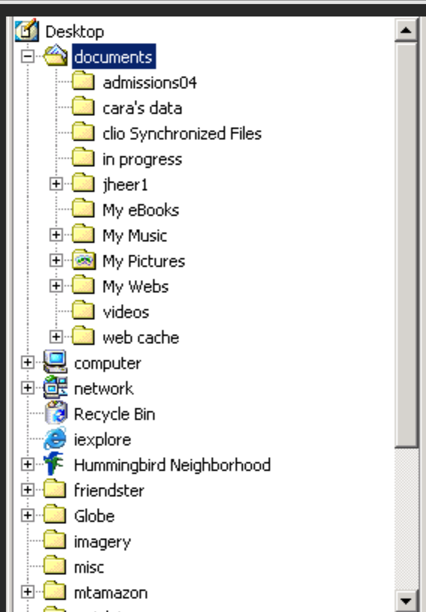
- Represent hierarchy by enclosure

Layering

- Layering and alignment

Tree layout is fast: O(n) or O(n log n), enabling real-time layout for interaction

Indentation



Items along vertically spaced rows

Indentation shows parent/child relationships

Often used in interfaces

Breadth/depth contend for space

Often requires scrolling

Node-Link Diagrams

Nodes distributed in space, connected by straight/curved lines

Use 2D space to break apart breadth and depth

Space used to communicate hierarchical orientation

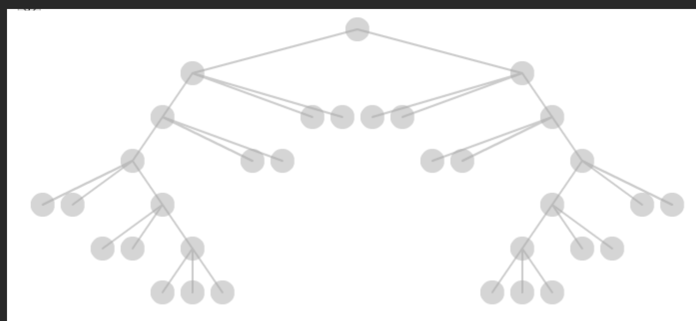
- Typically towards authority or generality

Basic Recursive Approach

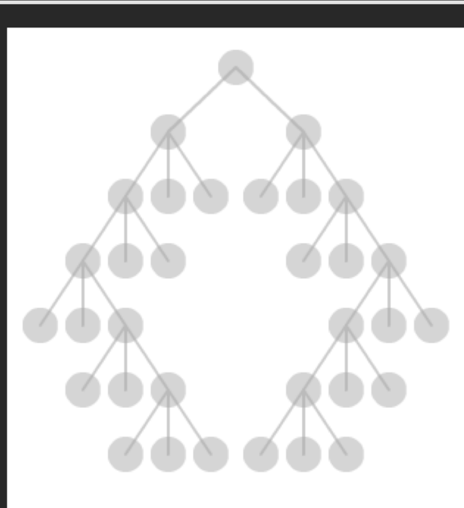
Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension

Problem: exponential growth of breadth



Reingold & Tilford' s Tidier Layout



Goal: maximize density and symmetry.

Originally for binary trees, extended by Walker to cover general case.

This extension was corrected by Buchheim et al. to achieve a linear time algorithm

Reingold-Tilford Layout

Design concerns

Clearly encode depth level

No edge crossings

Isomorphic subtrees drawn identically

Ordering and symmetry preserved

Compact layout (don't waste space)

Reingold-Tilford Algorithm

Linear algorithm – starts with bottom-up (postorder) pass

Set Y-coord by depth, arbitrary starting X-coord

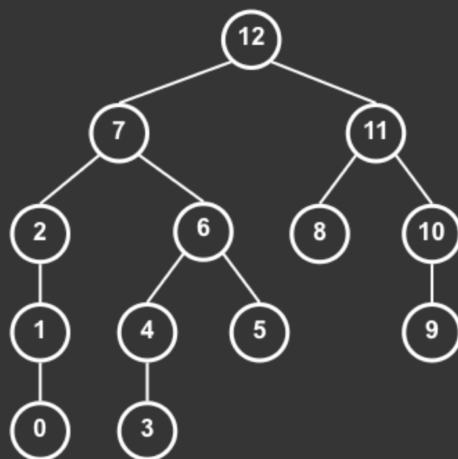
Merge left and right subtrees

- Shift right as close as possible to left
  - Computed efficiently by maintaining subtree contours
- "Shifts" in position saved for each node as visited
- Parent nodes are centered above their children

Top-down (preorder) pass for assignment of final positions

- Sum of initial layout and aggregated shifts

Reingold-Tilford Algorithm



Reingold-Tilford Algorithm

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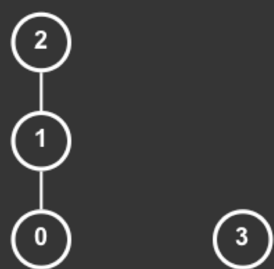

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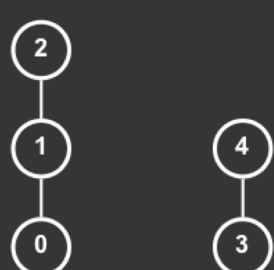

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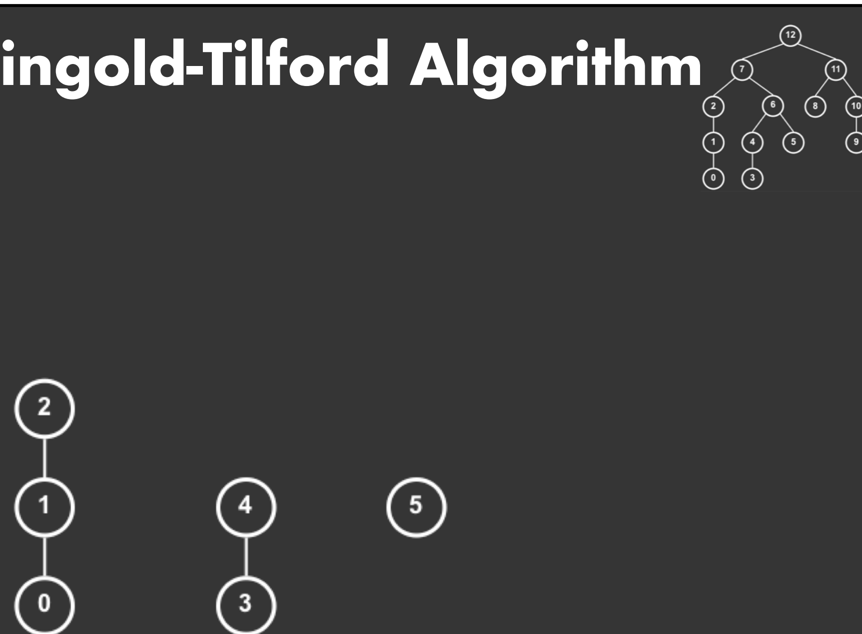

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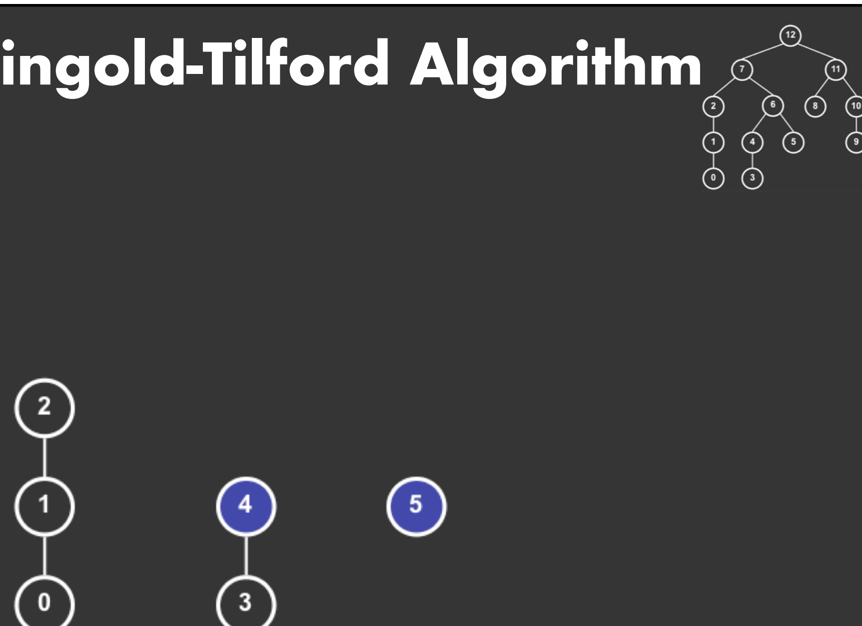

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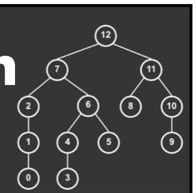

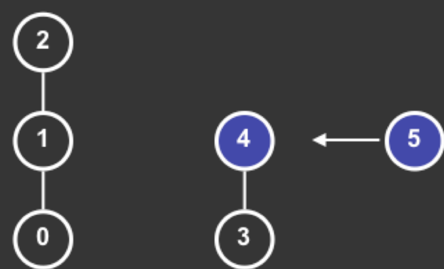

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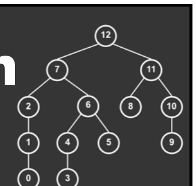

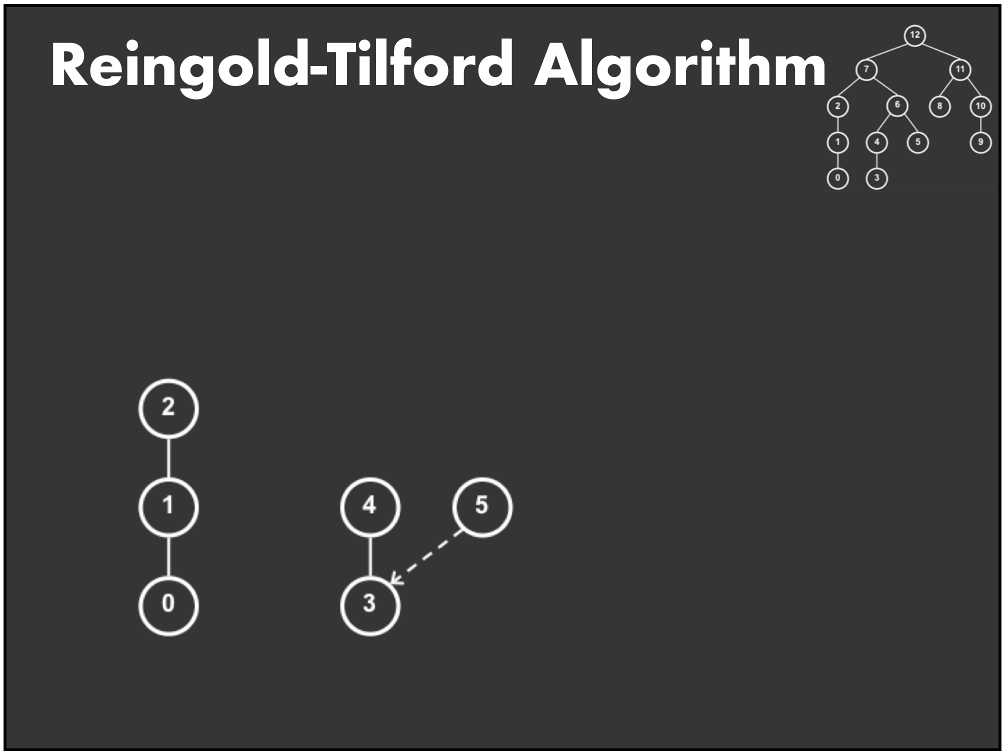
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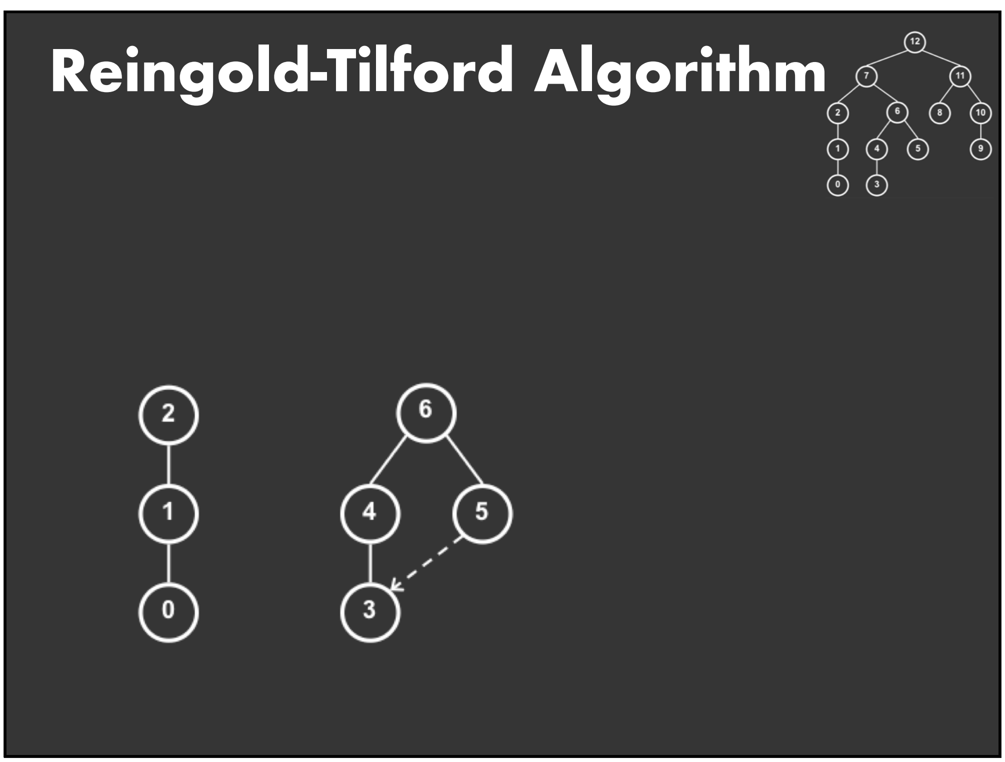
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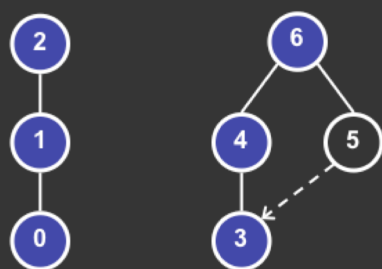

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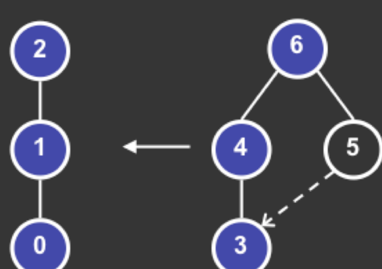

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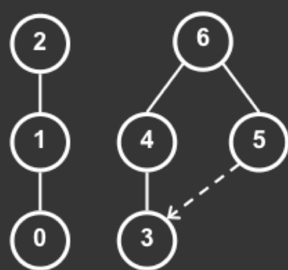

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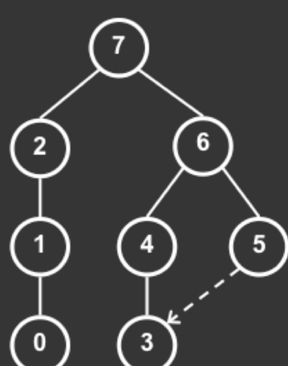

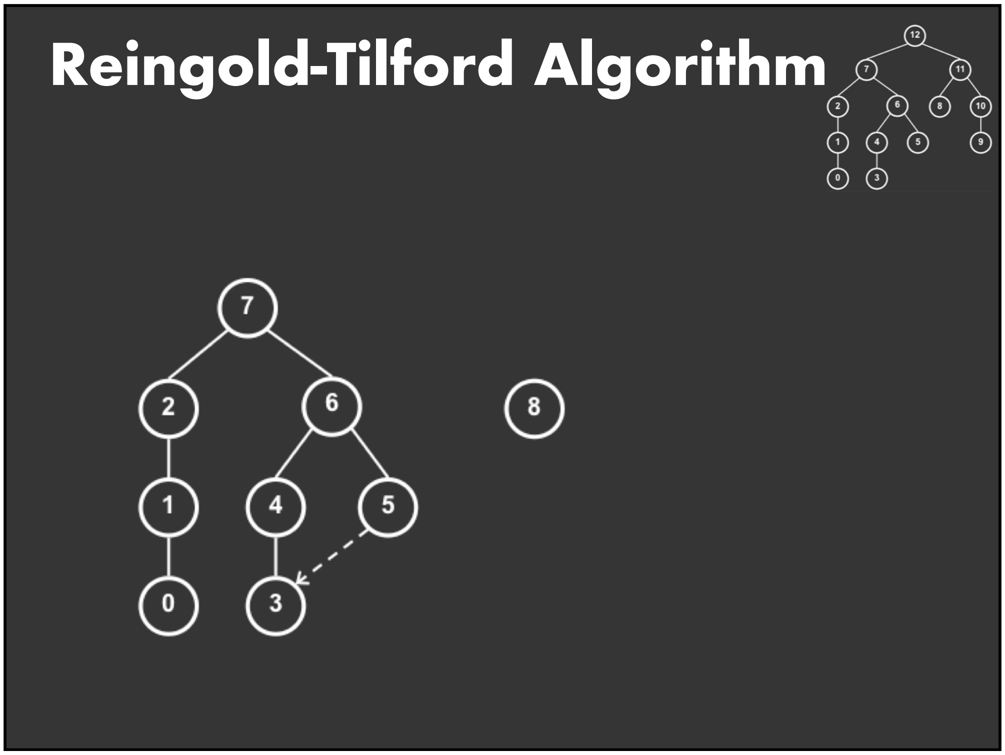
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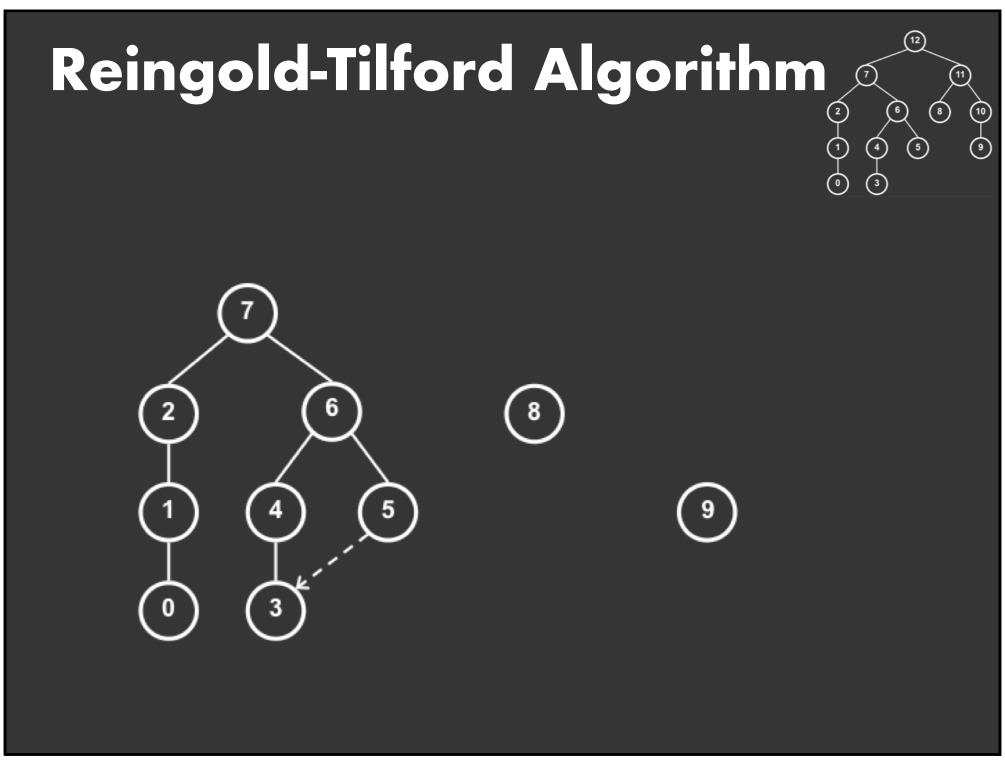
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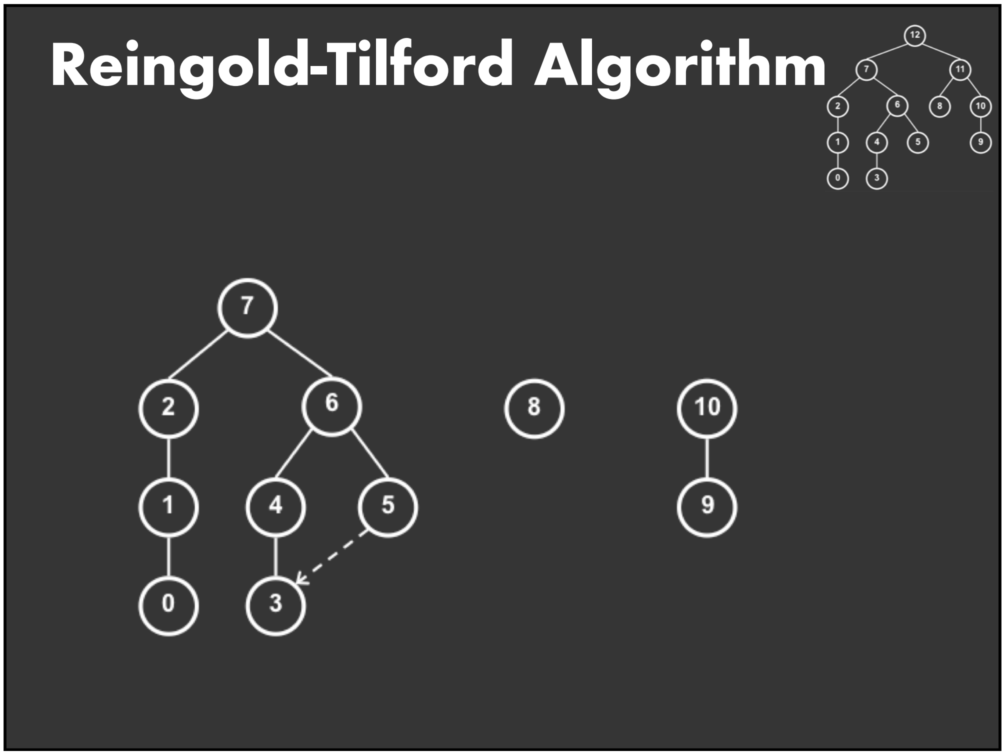
Reingold-Tilford Algorithm
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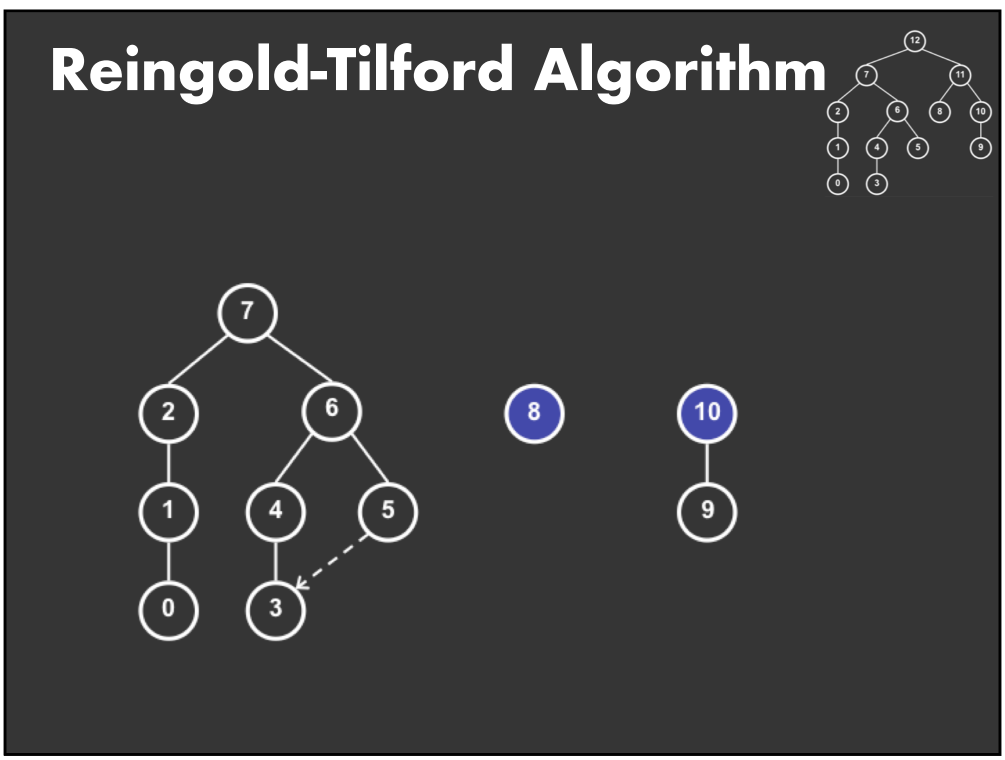
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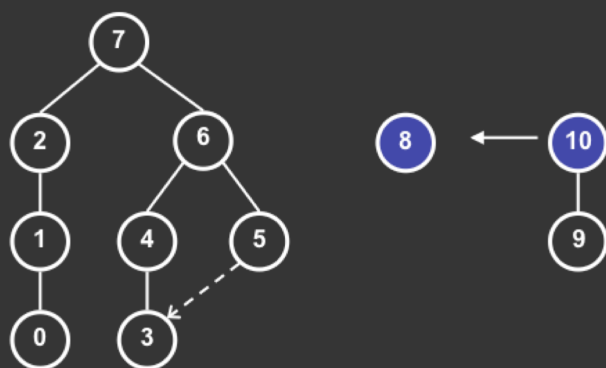
Reingold-Tilford Algorithm



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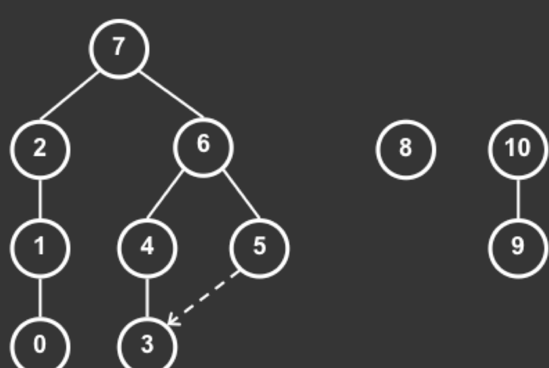

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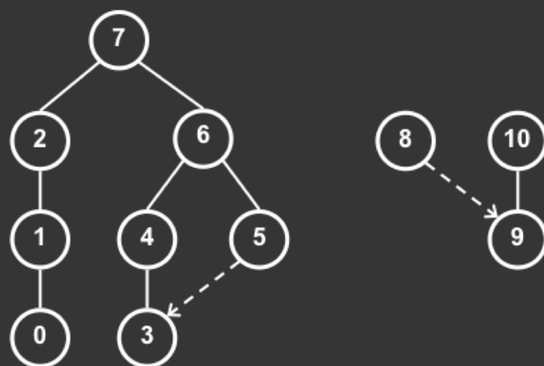

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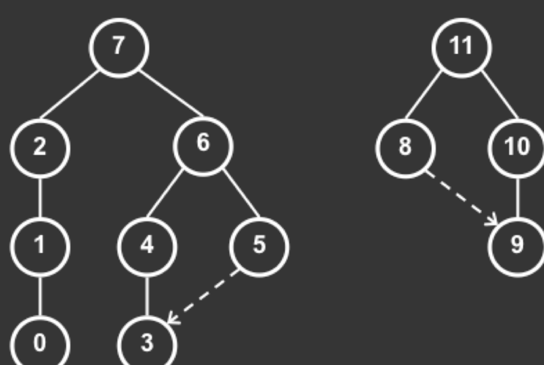

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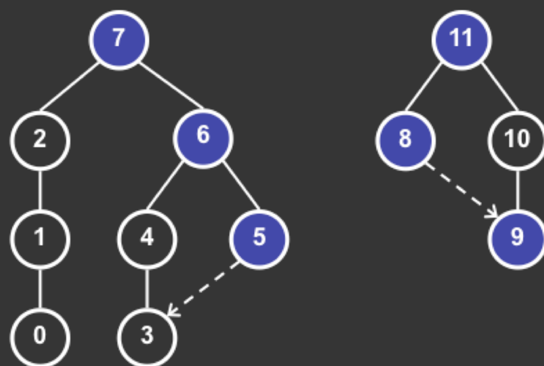

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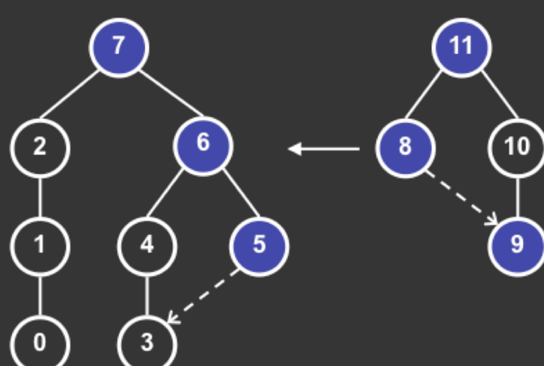

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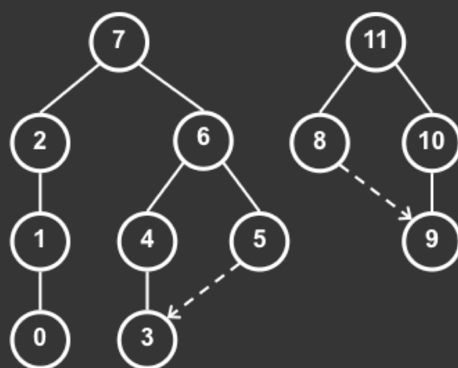

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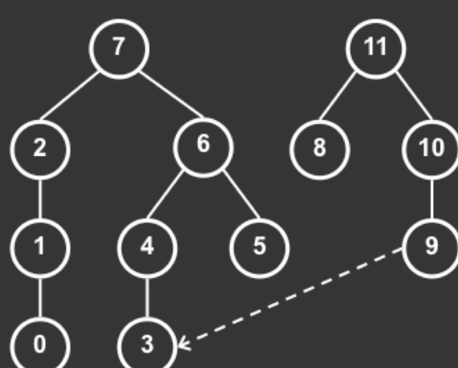

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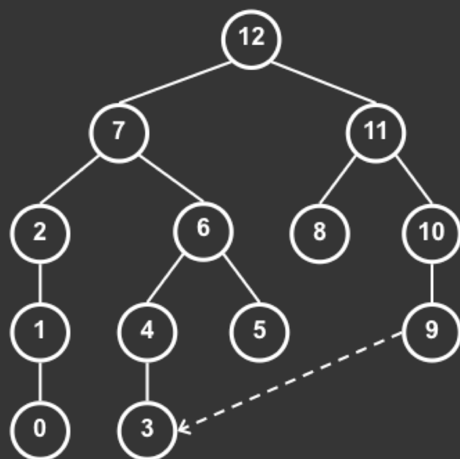
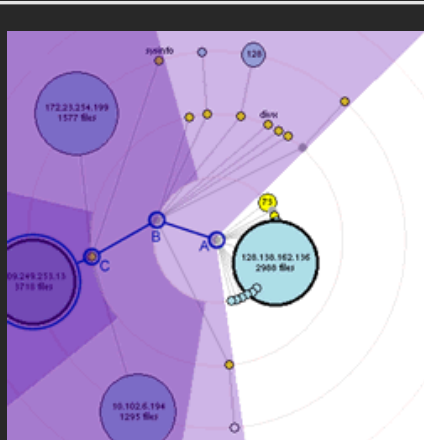
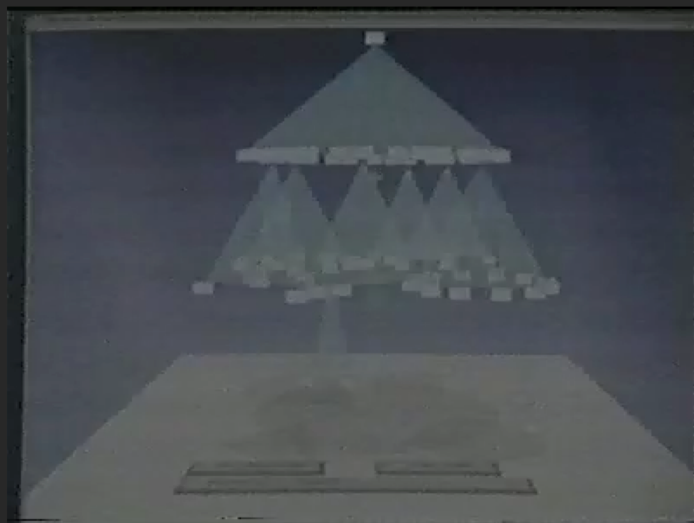


Radial Layout



- Node-link diagram in polar coords
- Radius encodes depth root at center
- Angular sectors assigned to subtrees (recursive approach)
- Reingold-Tilford approach can also be applied here

Circular Drawing of Trees in 3D



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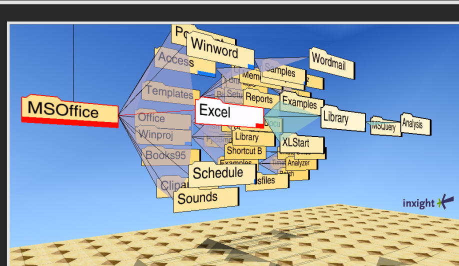
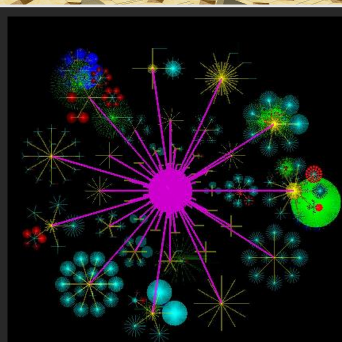


Cone trees – 3D layout



Balloon Trees = 2D Cone Trees

Not just flattening – circles must not overlap

Problems with Node-Link Diagrams

Scale

Tree breadth often grows exponentially
Even with tidier layout, quickly run out of space

Possible solutions

Filtering
Focus+Context
Scrolling or Panning
Zooming
Aggregation

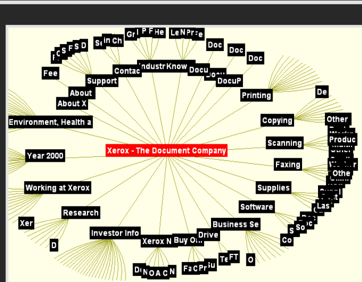
Visualizing Large Hierarchies

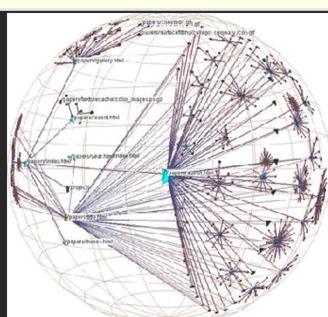
Indented Layout

Reingold-Tilford Layout

MC Escher, *Circle Limit IV*

Hyperbolic Layout



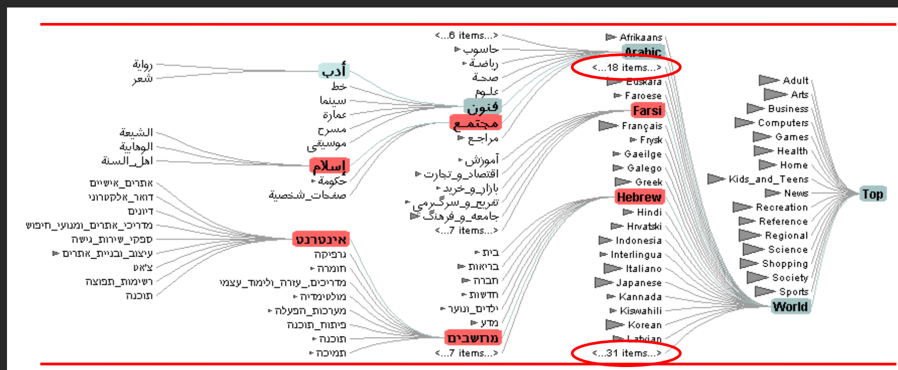


Layout in hyperbolic space, then project on to Euclidean plane

Why? Like tree breadth, the hyperbolic plane expands exponentially

Also computable in 3D, projected into a sphere

Degree-of-Interest Trees [AVI 04]

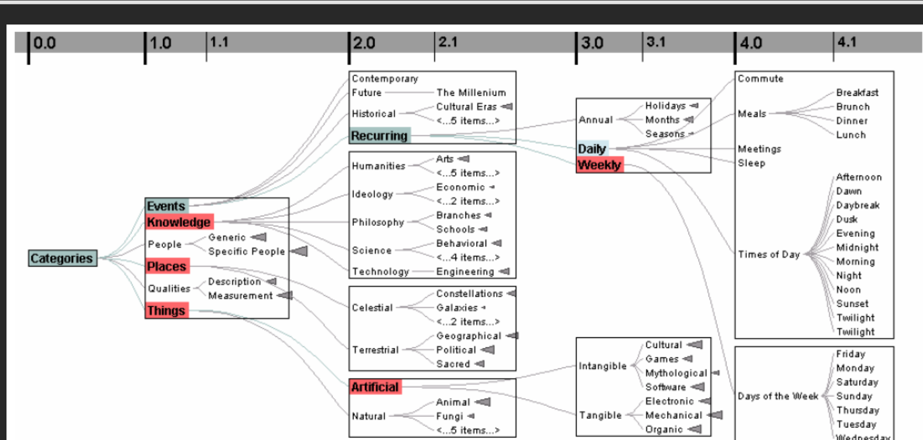


Space-constrained, multi-focal tree layout

https://www.youtube.com/watch?v=RTQ0N4QY0yc

https://bl.ocks.org/mbostock/4339083

Degree-of-Interest Trees



Cull "un-interesting" nodes on a per block basis until all blocks on a level fit within bounds

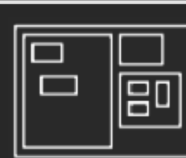
Center child blocks under parents

https://www.youtube.com/watch?v=RTQ0N4QY0yc

https://bl.ocks.org/mbostock/4339083

Enclosure Diagrams

Encode structure using spatial enclosure
Popularly known as TreeMaps



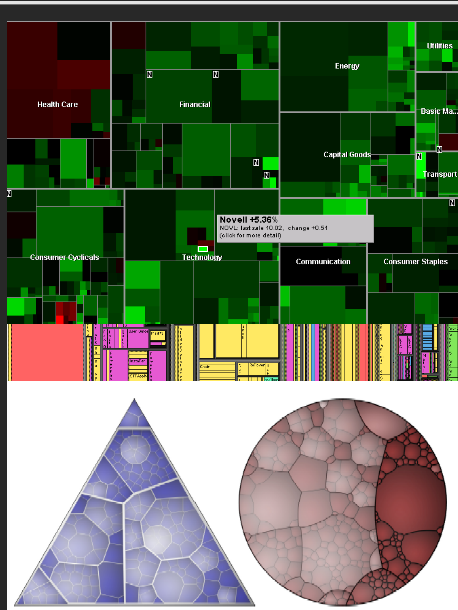
Benefits

- Provides a single view of an entire tree
- Easier to spot large/small nodes

Problems

- Difficult to accurately read depth

TreeMaps



Recursively fill space based on node size

Enclosure signifies hierarchy

Additional measures to control aspect ratio of cells

Often uses rectangles, but other shapes are possible, e.g., iterative Voronoi tesselation.

https://finviz.com/map.ashx

Layered Diagrams

Signify tree structure using

Layering

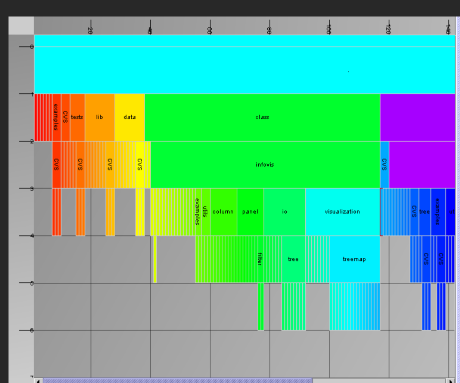
Adjacency

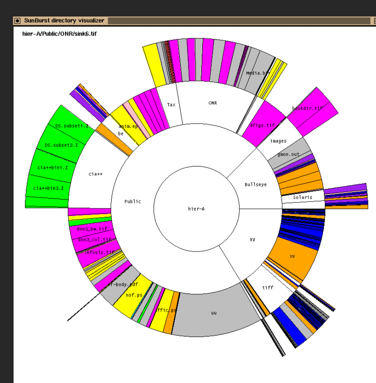
Alignment

Involves recursive sub-division of space

Can apply the same set of approaches as in node-link layout

Icicle and Sunburst Trees

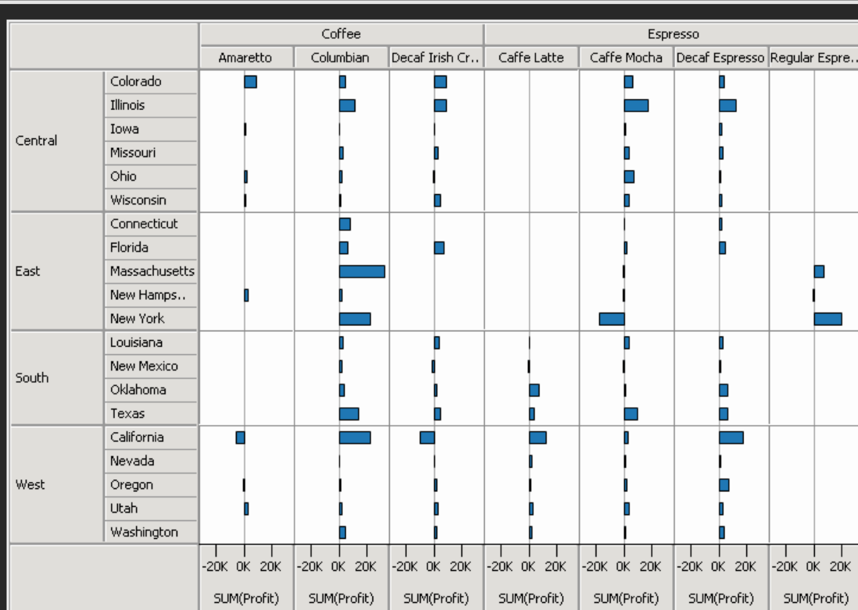




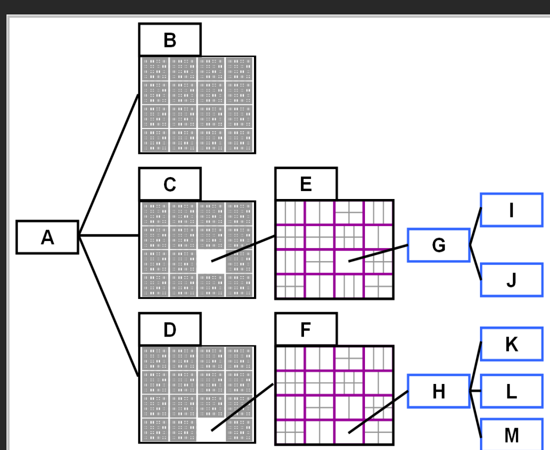
Higher-level nodes get a larger layer area, whether that is horizontal or angular extent

Child levels are layered, constrained to parent' s extent

Layered Tree Drawing



Hybrids are also possible…



"Elastic Hierarchies"

Node-link diagram with treemap nodes

Graph Visualization

Approaches to Graph Drawing

Direct calculation using graph structure

- Tree layout on spanning tree
- Hierarchical layout
- Adjacency matrix layout

Optimization-based layout

- Constraint satisfaction
- Force-directed layout

Attribute-driven layout

- Layout using data attributes, not linkage

Spanning Tree Layout

Many graphs are tree-like or have useful spanning trees

Websites, Social Networks

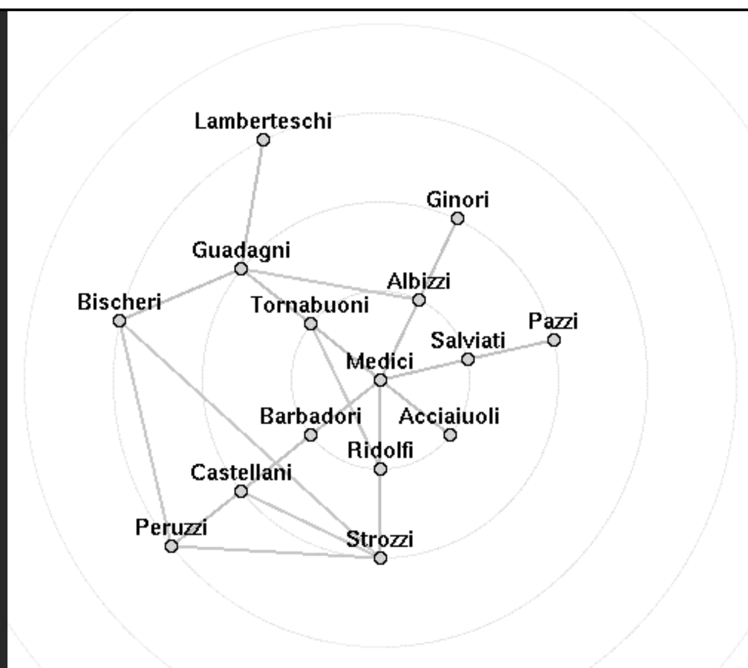
Use tree layout on spanning tree of graph

Trees created by BFS / DFS

Min/max spanning trees

Fast tree layouts allow graph layouts to be recalculated at interactive rates

Heuristics may further improve layout

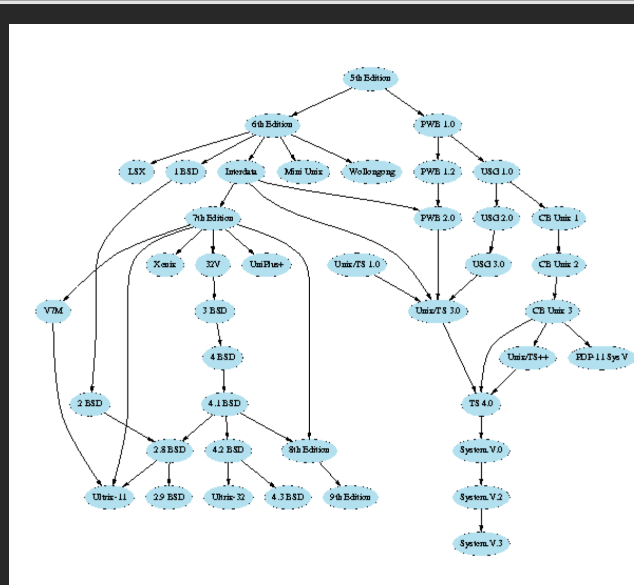


Spanning tree layout may result in arbitrary parent node

Sugiyama-style graph layout

Evolution of the UNIX operating system

Hierarchical layering based on descent



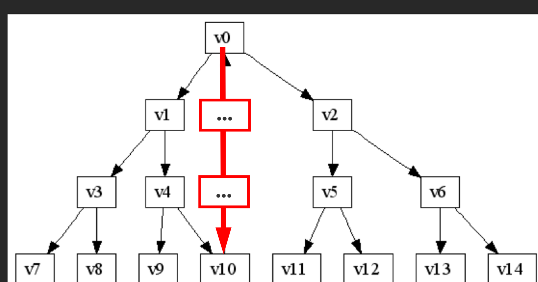
Sugiyama-style graph layout

Layer 1

Layer 2

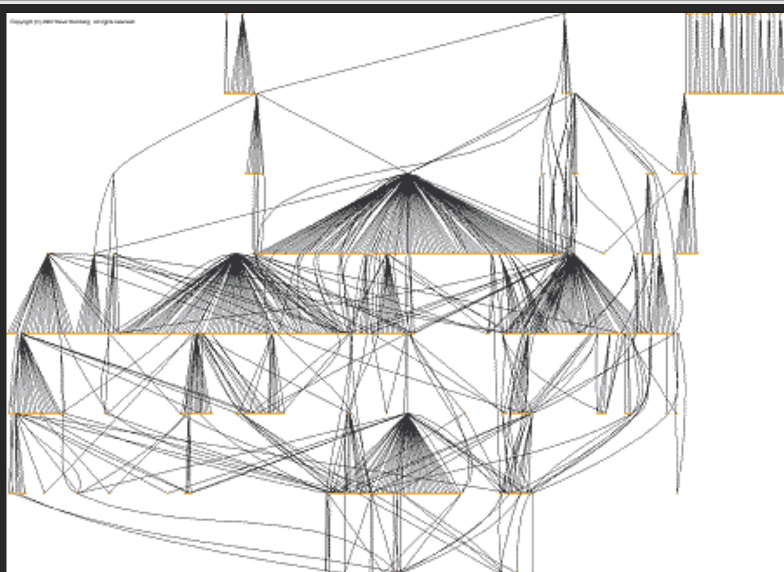
Layer 3

Layer 4



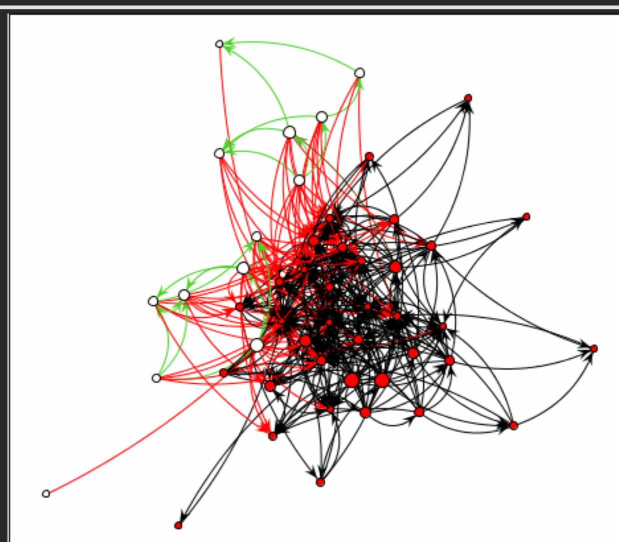
Reverse some edges to remove cycles
Assign nodes to hierarchy layers → Longest path layering
    Create dummy nodes to "fill in" missing layers
Arrange nodes within layer, minimize edge crossings
    Route edges – layout splines if needed

Hierarchical graph layout

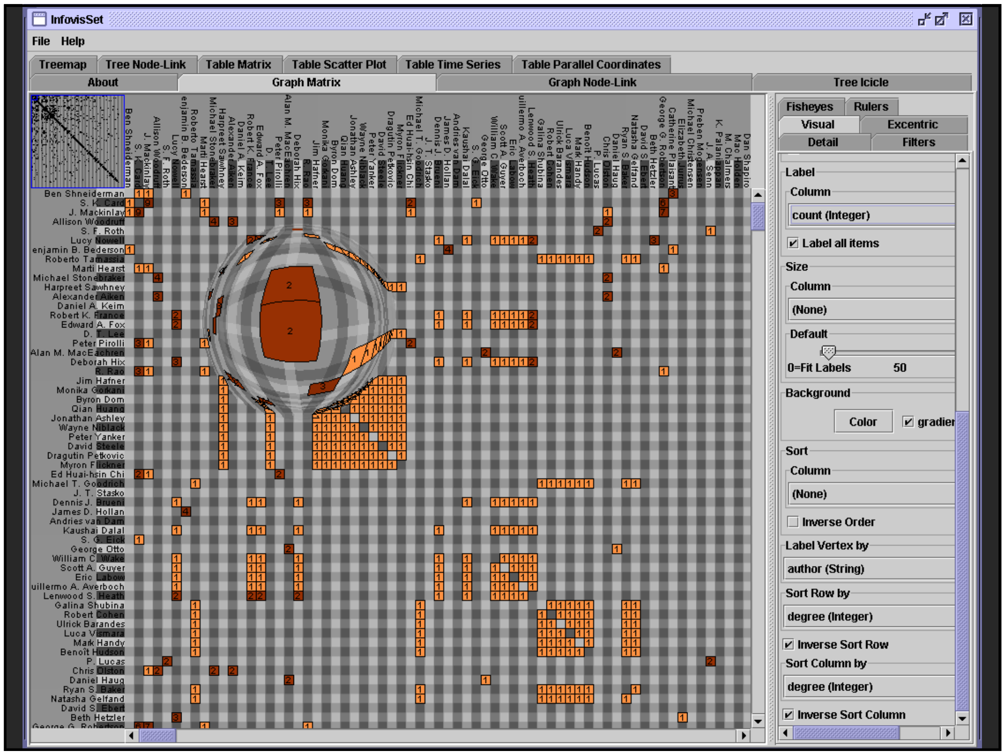


Gnutella network

Limitations of Node-Link Layout



Edge-crossings and occlusion



Optimization Techniques

Treat layout as an *optimization problem*

Define layout using a set of *constraints*:
equations the layout should try to obey

Use optimization algorithms to solve

Common approach for undirected graphs

***Force-Directed Layout* most common**

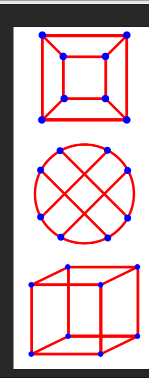
Can also introduce directional constraints

***DiG-CoLa* (Di-Graph Constrained Optimization Layout) [Dwyer 05]**

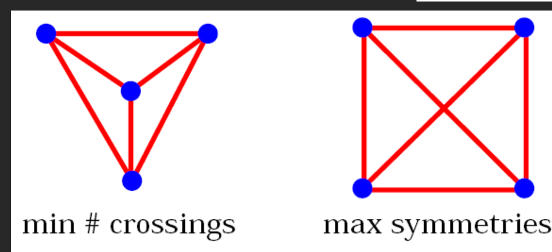
Optimizing “Aesthetic” Constraints

Minimize edge crossings
Minimize area
Minimize line bends
Minimize line slopes
Maximize smallest angle between edges
Maximize symmetry

but, can’t do it all



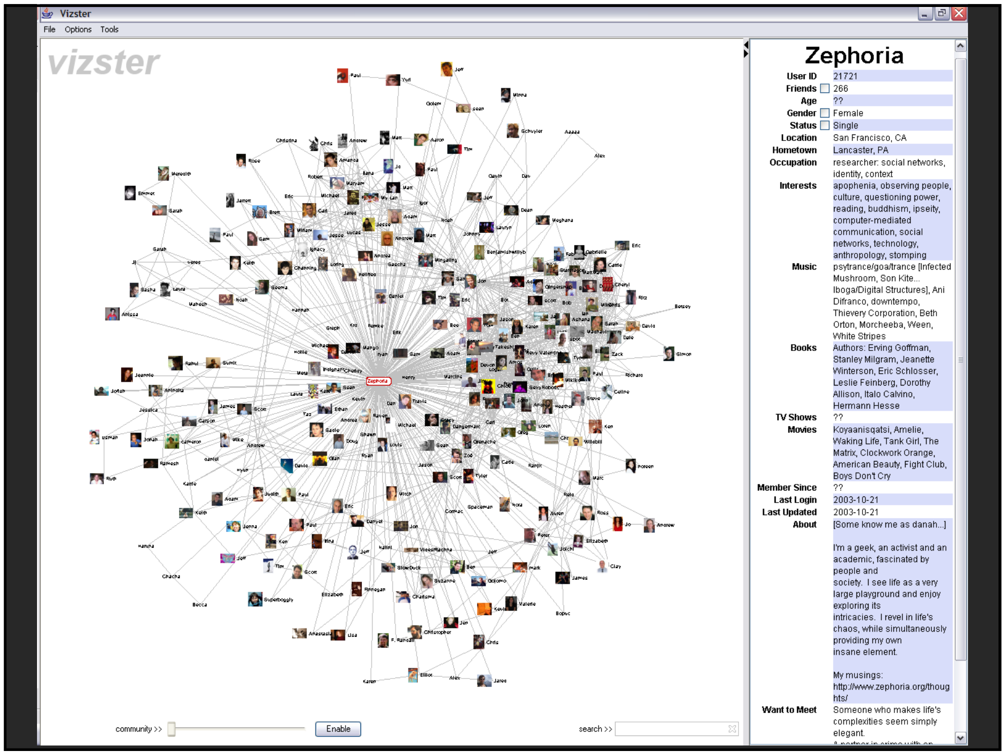
Optimizing these criteria is often NP-Hard, requiring approximations



Force-Directed Layout

Edges = springs $F = -k * (x - L)$
Nodes = charged particles $F = G*m_1*m_2 / x^2$

Repeatedly calculate forces, update node positions
- Naïve approach $O(N^2)$
- Speed up to $O(N \log N)$ using quadtree or k-d tree
- Numerical integration of forces at each time step



Constrained Optimization Layout

Minimize stress function

$$\text{stress}(X) = \Sigma_{i<j}\, w_{ij}\, (\, \|X_i - X_j\| - d_{ij}\,)^2$$

- X: node positions, d: optimal edge length,
- w: normalization constants
- Use global (*majorization*) or localized (*gradient descent*) optimization

→ **Says: Try to place nodes d_{ij} apart**

Add hierarchy ordering constraints

$$E_H(y) = \Sigma_{(i,j)\in E}\, (\, y_i - y_j - \delta_{ij}\,)^2$$

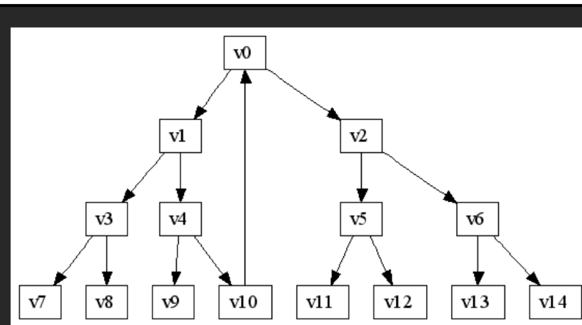
- y: node y-coordinates
- δ : edge direction (e.g., 1 for i→j, 0 for undirected)

→ **Says: If i points to j, it should have a lower y-value**

[Dwyer 05]

Sugiyama layout (dot)

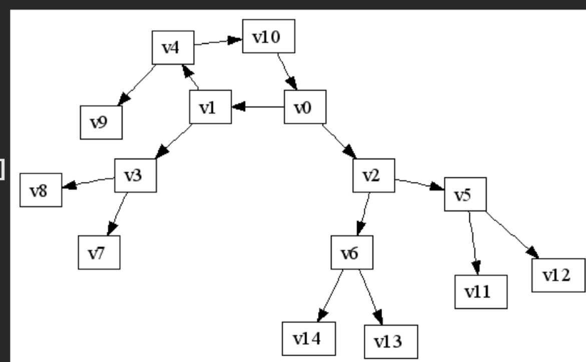
Preserve tree structure



DiG-CoLa method

Preserve edge lengths

[Dwyer 05]



Attribute-Driven Layout

Large node-link diagrams get messy!

Is there additional structure we can exploit?

Idea: Use data attributes to perform layout

- e.g., scatter plot based on node values

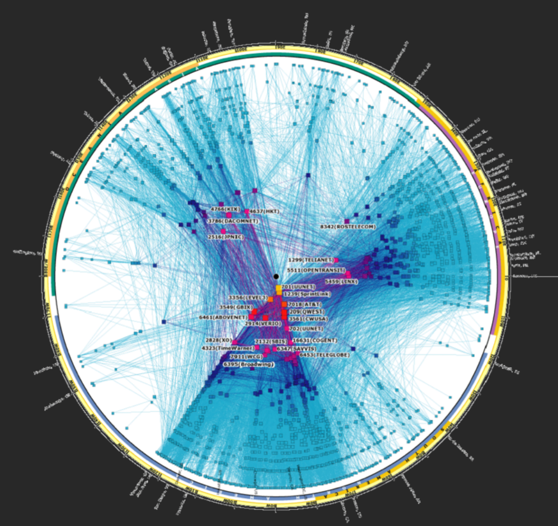
Dynamic queries and/or brushing can be used to explore connectivity

Attribute-Driven Layout

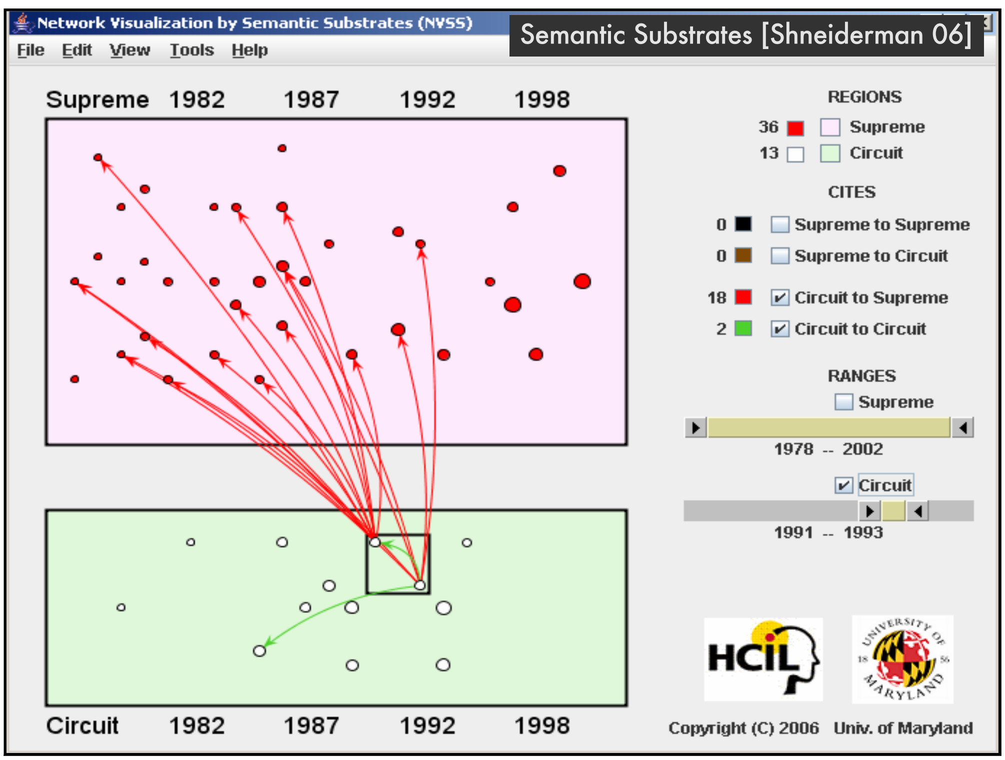
The "Skitter" Layout
- Internet Connectivity
- Radial Scatterplot

Angle = Longitude
- Geography

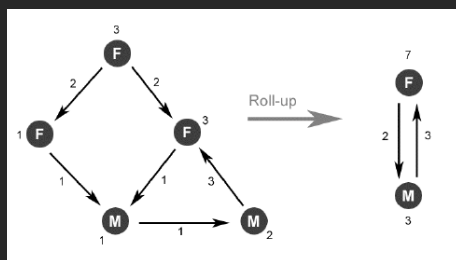
Radius = Degree
- # of connections
- (a statistic of the nodes)

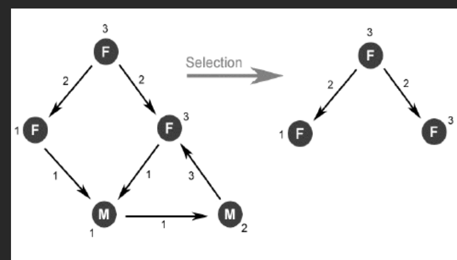


Semantic Substrates [Shneiderman 06]



PivotGraph [Wattenberg 2006]





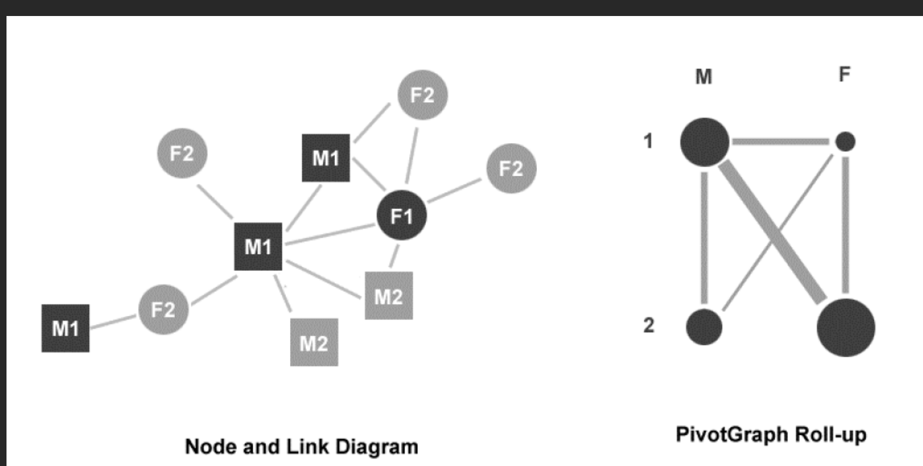
Roll-Up

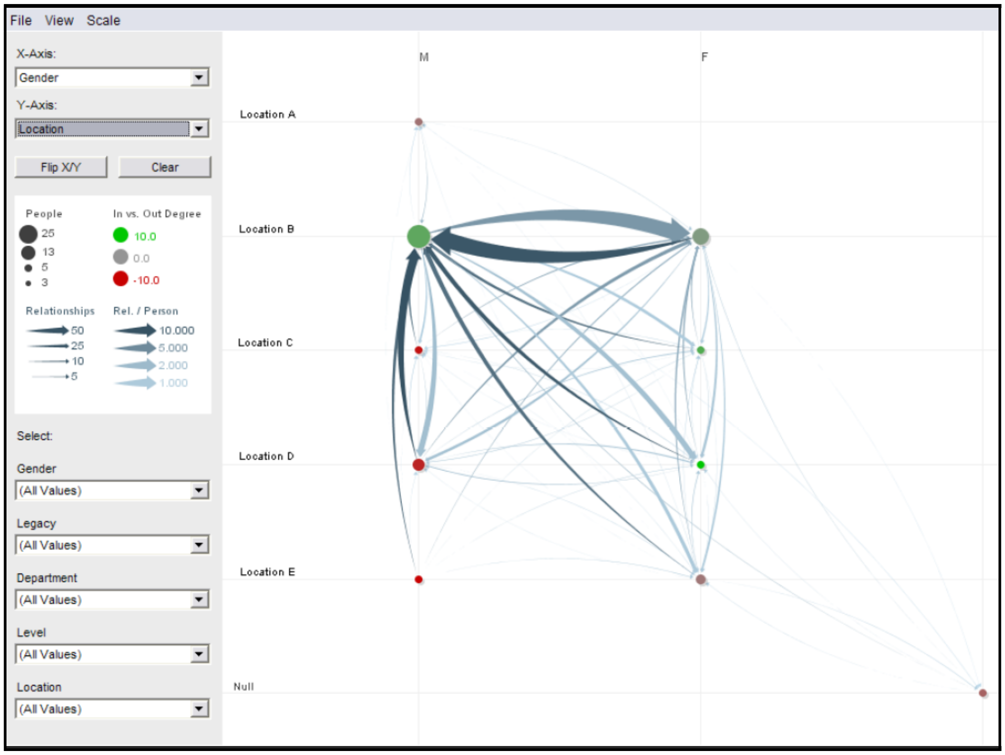
Aggregate items with matching data values

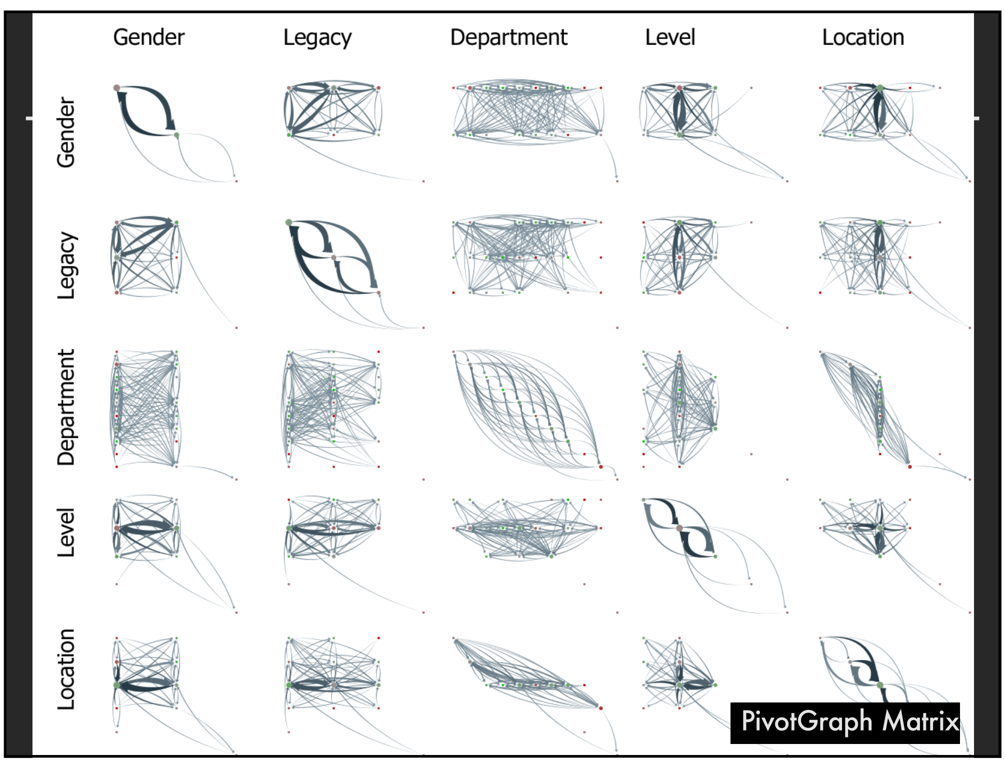
Selection

Filter on data values

PivotGraph




File View Scale
X-Axis:
Gender
Y-Axis:
Location
Flip X/Y
Clear
People
25
13
5
3
In vs. Out Degree
10.0
0.0
-10.0
Relationships
50
25
10
5
Rel. / Person
10.000
5.000
2.000
1.000
Select:
Gender
(All Values)
Legacy
(All Values)
Department
(All Values)
Level
(All Values)
Location
(All Values)
M
F
Location A
Location B
Location C
Location D
Location E
Null

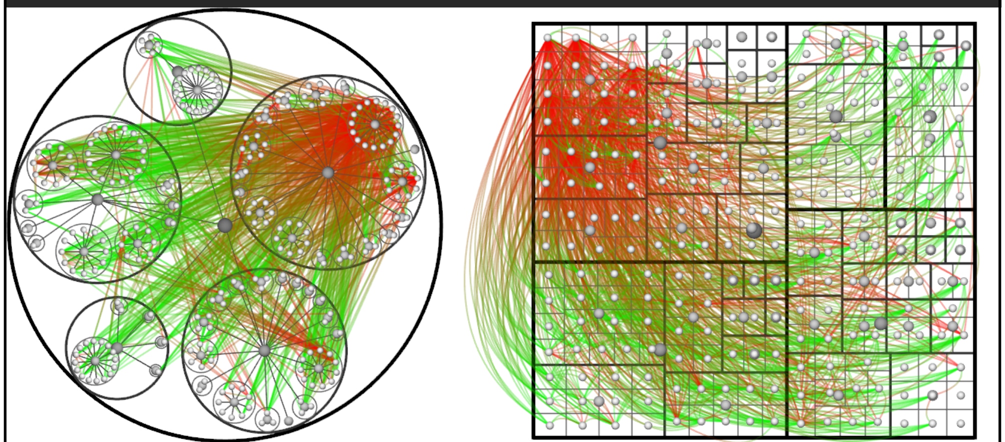

Gender
Legacy
Department
Level
Location
Gender
Legacy
Department
Level
Location
PivotGraph Matrix

Limitations of PivotGraph

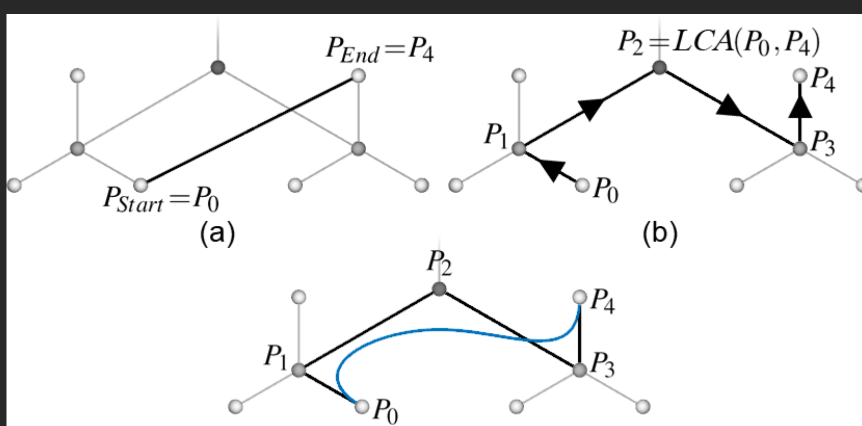
Only 2 variables (no nesting as in Tableau)
Doesn't support continuous variables
Multivariate edges?

Hierarchical Edge Bundles

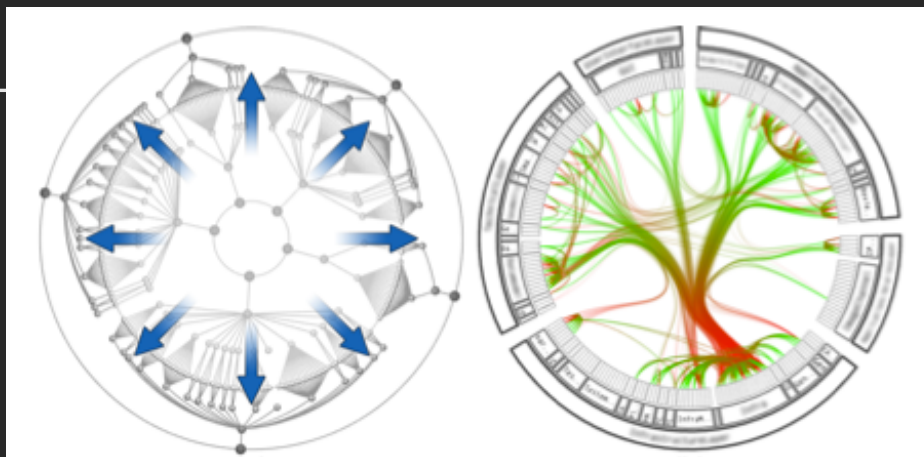
Trees with Adjacency Relations


(a) (b)

Bundle Edges along Hierarchy

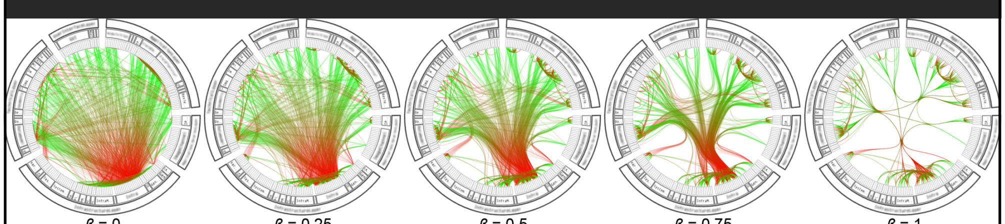


(a) (b) (c)

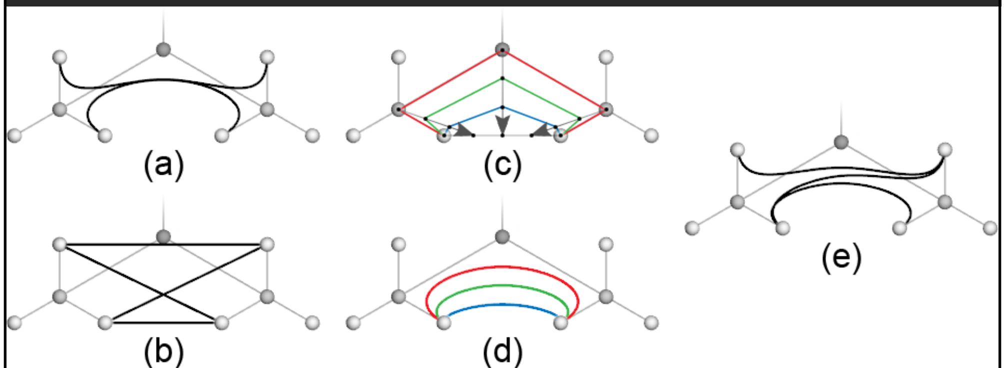


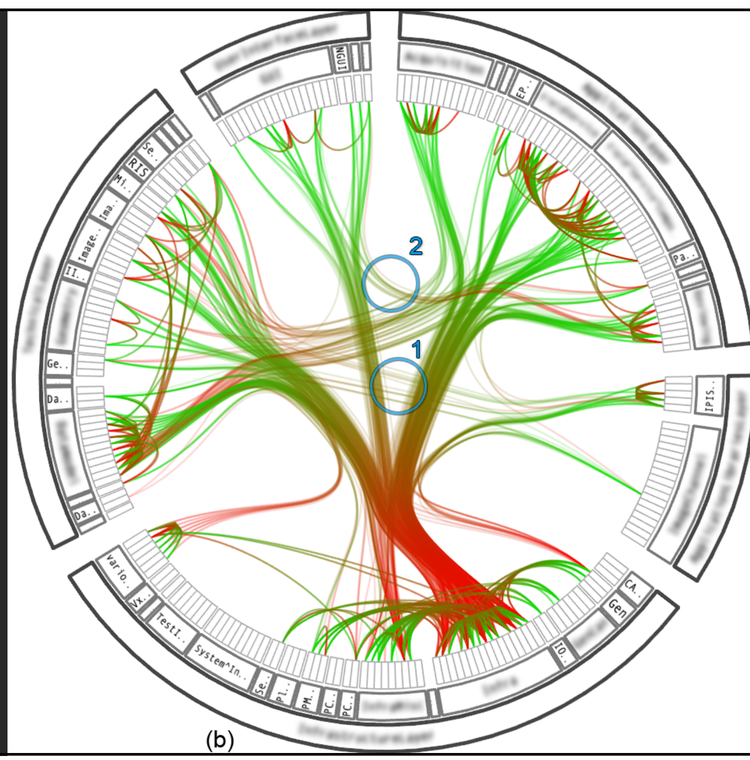
Use radial tree layout for inner circle
Mirror to outside
Replace inner tree with hierarchical edge bundles

Increasing Edge Tension



Configuring Edge Tension





Summary

Tree Layout

Indented / Node-Link / Enclosure / Layers

How to address issues of scale?

- Filtering and Focus + Context techniques

Graph Layout

Tree layout over spanning tree

Hierarchical "Sugiyama" Layout

Optimization (Force-Directed Layout)

Attribute-Driven Layout